Radiative Transfer in a Cloudy Atmosphere and Its Parameterization

5.1. INTRODUCTION

In the early stages of cloud modeling, modelers ignored the effects of radiative transfer. This is largely because the emphasis was on the simulation of individual convective clouds. For convective time scales of the order of 30 minutes to 1 hour, radiative heating rates are of little importance. However, as cloud modeling moved to the simulation of stratocumulus clouds, fogs, middle and high clouds, and to cloud processes on the mesoscale, where time scales are on the order of a day, cloud modelers have begun to consider radiative transfer processes. Moreover, with recent emphasis on global climate change, interest in cloud radiation feedback increased dramatically because it is widely recognized that clouds influence global albedo and the longwave radiation budget, and contribute to global temperature changes. Even tradewind cumulus, which has a short cloud life, exhibits, on average, an important global albedo effect. Another impetus for cloud radiation studies is related to remote sensing in which instantaneous radiances are observed.

As with turbulent transport and cloud microphysical processes, cloud modelers must make a number of compromises in the design of a radiative transfer model (i.e. they must formulate the radiative transfer equations in a simplified parameterized form). Similar to atmospheric motions that span a broad range of eddy sizes and cloud hydrometeors that span a broad range of particle sizes, atmospheric radiation covers a broad spectrum of radiation frequencies, wavelengths, or wave numbers. The sun, for example, emits radiation approximately as a blackbody having temperatures between 6000 and 5700 K, which peaks in intensity at a wavelength of 0.470 µm but which spans the range from less than 0.2 µm to greater than 1.8 µm, but 50% of all solar energy is between 0.3 and 0.7 µ (see Fig. 5.1). In contrast, the radiation energy emitted by the earth corresponds approximately to blackbody radiation at a temperature of about 250 K. Thus, a combination of radiation emitted by the sun and the earth spans a range from less than 0.2 µm to greater than 50 µm. However, the spectrum of radiation emitted by the sun and by the earth exhibits...
very little overlap. For this reason we refer to radiation emitted by the sun as shortwave radiation and radiation emitted by the earth and its atmosphere as longwave radiation. The two regions are separated arbitrarily at around 4 µm which is sufficient when considering the dynamics of clouds where energy transfer is of primary importance. As we shall see later, this distinction allows some simplifications in the formulation of radiative transfer theories.

The most important consideration, as far as the dynamics of clouds is concerned, is that the net heating rate at various levels in a cloud is a consequence of the attenuation of atmospheric radiation. The net heating rate is a function of the net radiative flux divergence,

$$\frac{\partial \theta}{\partial t} = \left(\frac{1}{\rho_0 c_p}\right)(\partial F_N/\partial z),$$  \hspace{1cm} (5.1)

where $F_N$ is the difference between downward and upward fluxes and has units of watts per square meter. In Eq. (5.1), for simplification, we consider only vertical flux divergences. In some cloud modeling problems, however, it may be desirable to consider horizontal radiative flux divergence as well (Marshak and Davis, 2005). An example is a valley fog located between two radiating valley sides, or radiative fluxes passing through a field of cumuli. Nonetheless, for most cloud modeling applications, consideration of vertical radiative flux divergences is sufficient. The net vertical flux $F_N$ is the difference between downward and upward fluxes, or

$$F_N = F \downarrow - F \uparrow.$$  \hspace{1cm} (5.2)

The upward and downward fluxes, in turn, represent the fluxes integrated over all wavelengths and averaged over upward and downward looking hemispheres.
A divergence of radiative flux is caused by a combination of differential extinction of radiation and thermal emission. Extinction of radiation is a result of absorption and of scattering of radiant energy. The absorptance $A_{\lambda}$ represents the fraction of incoming radiation absorbed in a layer of the atmosphere. The reflectance $Re_{\lambda}$ and transmittance $Tr_{\lambda}$ simply represent the fractions of incoming radiation which are scattered out of the primary beam, and transmitted through a layer of the atmosphere, respectively. Note that all three processes vary with the wavelength of radiation. As a result of conservation of energy,

$$ A_{\lambda} + Re_{\lambda} + Tr_{\lambda} = 1. \quad (5.3) $$

Light scattering without absorption is associated with redistribution of energy, a process which leads to a diffuse radiation field and is also called conservative scattering.

Absorption results in a change in the internal energy or temperature of the medium. In the atmosphere this change is usually a change in internal energy or temperature. If the incident energy as well as the scattered and transmitted energy remain constant for a time, the internal energy of the system will remain unchanged. A radiative equilibrium is established in which as much radiation is being emitted as is being absorbed. However, in conditions of local thermodynamic equilibrium, the emitted radiation is in thermal equilibrium with the source level. If the atmosphere were a pure blackbody (i.e. $A_{\lambda} = 1$ for all wavelengths), then the total emitted radiative flux would be

$$ F = \sigma T^4, \quad (5.4) $$

where $\sigma$ is the Stefan-Boltzmann constant. According to Blevin and Brown (1971), $\sigma = (5.66961 \pm 0.0075) \times 10^{-8}$ W m$^{-2}$ K$^{-4}$. Equation (5.4) can be obtained by integrating the Planck radiation distribution function over all wavelengths (see any basic radiation physics text for a definition of the Planck function). The atmosphere does not, however, behave as a blackbody. Therefore, the amount of energy emitted by the absorbing atmosphere is given by

$$ F = \varepsilon \sigma T^4, \quad (5.5) $$

where $\varepsilon$ represents the emittance of the atmosphere. The emittance is the ratio of the flux emitted by a body to the flux emitted by a blackbody at the same temperature. We shall see later that the effective emittance of a cloudy atmosphere varies with the liquid-water content and particle spectra in clouds.

### 5.2. ABSORPTANCE, REFLECTANCE, TRANSMITTANCE, AND EMITTANCE IN THE CLEAR ATMOSPHERE

As can be seen in Fig. 5.1, in a cloud-free and aerosol-free atmosphere, the primary absorbers of shortwave radiation are ozone and water vapor. Aerosols,
FIGURE 5.2 Atmospheric spectrum obtained with a scanning interferometer on board the Nimbus 4 satellite. The interferometer viewed the earth vertically as the satellite was passing over the North African desert. (After Hanel et al. (1972), cited in Paltridge and Platt (1976))

particularly soot or carbonaceous particles, also contribute to a lesser extent to absorption. At wavelengths shorter than 0.3 μm, oxygen and nitrogen absorb nearly all incoming solar radiation in the upper atmosphere. However, in the important region of visible radiation between 0.3 and 0.7 μm, little gaseous absorption occurs. Only weak absorption by ozone takes place in this spectral range. This is fortunate because these are the wavelengths in which the solar radiation peaks. At wavelengths of less than 0.7 μm, Rayleigh scattering of shortwave radiation back to space depletes the available flux. At longer wavelengths, absorption in various water vapor bands is quite pronounced. Some weak absorption by carbon dioxide and ozone also occurs at wavelengths greater than 0.7 μm. Absorption by carbon dioxide is a subject of considerable attention because of its importance for global warming. Cloud processes are important to climate change because positive or negative temperature changes may alter cloud cover and cloud microphysics, which can feedback on greenhouse gas warming.

Absorption of longwave radiation also occurs mainly in a series of bands. The principal absorbers of longwave or infrared (IR) radiation are water vapor, carbon dioxide, and ozone. Figure 5.2 illustrates the banded character of absorption in the IR region. Figure 5.2 represents the IR spectrum obtained by a scanning interferometer looking downward from a satellite over a desert region. Strong absorption by CO₂ at a wavelength band centered at 14.7 μm is shown by the emission of radiance at the temperature of 220 K. The stratosphere contributes mainly to the peak of this absorption band, with warmer
tropospheric contributions occurring across the broader part of the absorption band. Water vapor absorption bands at 1.4, 1.9, 2.7, and 6.3 µm, and greater than 20 µm, cause emissions corresponding to mid-tropospheric temperatures. Little absorption is evident in the region called the “atmospheric window” between 8 and 14 µm. Here the radiance corresponds to the surface temperature of the desert, except for a slight depression in magnitude due to departures of the emittance of sand from unity. A distinct ozone absorption band is evident in the region of 9.6 µm in the middle of the window. Although not very evident in the figure, weak continuous absorption also occurs across the “window.” The intensity of the continuum absorption depends on water vapor pressure. This continuous absorption is due to the presence of clusters or dimers (H₂O)_2 of water vapor molecules.

A detailed knowledge of the absorption spectra in the IR region is essential to predicting the rate of cooling due to longwave radiative transfer. In the following sections, we first examine the interactions among cloud particles and radiation. Then we review some of the techniques for calculating radiative transfer in a cloudy atmosphere.

5.3. SHORTWAVE RADIATIVE TRANSFER IN A CLOUDY ATMOSPHERE

The interaction of solar radiation incident upon a cloud is complicated by the fact that not only must we concern ourselves with the impact of a spectrum of radiative frequencies on cloud absorption, reflection, and transmission, but also with the consequences of a spectrum of droplets or ice crystals on radiative transfer. In the absence of emission, the azimuthally averaged, horizontally homogeneous plane parallel, time-independent radiative transfer equation appropriate to a cloudy medium is as follows:

\[
\frac{dI(\tau, \mu)}{d\tau} = -I(\tau, \mu) + \frac{\tilde{\omega}_0}{2} \int_{-1}^{+1} \bar{p}(\tau, \mu, \mu') I(\tau, \mu') d\mu' + \frac{S_0}{4\pi} \bar{p}(\tau, \mu, \mu_0) e^{-\tau/\mu_0},
\]

where \( \tau \) is the optical depth, \( \omega_0 \) is the single-scattering albedo, \( \bar{p} \) is the scattering phase function, and \( S_0 \) is the solar flux associated with a collimated beam incident on the cloud top. All parameters in Eq. (5.6) are functions of the frequency of radiation \( \nu \) with the exception of \( \mu \) and \( \mu_0 \), where \( \mu \) is a function of the cosine of the zenith angle. The quantity \( I(\tau, \mu) \) is the radiance along an angle given by \( \mu \) through a cloudy layer defined by the optical depth \( \tau \). The extinction optical thickness, \( \tau \), is the sum of droplet scattering, droplet absorption, and gaseous absorption. Gaseous absorption is primarily due to water vapor in both a clear and a cloudy atmosphere. In fact, for some time it was thought that the major effect of clouds on shortwave radiation was to
increase the optical path length of water vapor absorption as a result of multiple scattering by cloud droplets. We shall see that absorption by liquid droplets plays an important role in the overall absorption in clouds. Cloud droplet absorption is described in terms of \( \tau_a \), the droplet absorption optical thickness. If we define \( \tau_s \), the scattering optical thickness, and \( \tau_g \), the gaseous absorption optical thickness, then the extinction optical thickness is given by

\[
\tau = \tau_s + \tau_a + \tau_g.
\]  

(5.7)

For any frequency one can calculate the optical thickness \( \tau(\nu) \) and thus obtain a single-scattering albedo

\[
\tilde{\omega}_0 = \tau_s/\tau.
\]  

(5.8)

Thus, \( \tilde{\omega}_0 = 1 \) for a nonabsorbing cloud and \( \tilde{\omega}_0 = 0 \) when scattering is negligible. The optical thickness varies only slightly with \( \nu \) and the average size of a droplet. The single-scattering albedo, on the other hand, varies strongly with both frequency (or wavelength) and droplet size. Figure 5.3 illustrates the variation in single-scattering albedo as a function of wavelength for a maritime cloud with a droplet concentration of 25 cm\(^{-3}\) (solid curve) and a mildly continental cloud with a droplet concentration of 200 cm\(^{-3}\) (dashed curve) for a given temperature and a liquid-water content of 0.33 gm\(^{-3}\). In a continental cloud, such as over the High Plains of the United States, where droplet concentrations are approximately 700 cm\(^{-3}\), or in polluted air masses, where droplet concentrations are approximately 2500 cm\(^{-3}\), the single-scattering albedo would be much greater than in a clean maritime air mass. This is because for a given liquid-water content, the higher droplet concentrations result in much smaller droplet sizes.
The scattering phase function $p(\tau, \mu, \mu')$ in Eq. (5.6) characterizes the angular distribution of the scattered radiation field. For spherical droplets, the function exhibits a strong peak in the forward direction and produces rainbow and glory effects in the backscattering directions; the sun has to be behind the observer to see a rainbow. As noted by Joseph et al. (1976), for some applications the phase function can be conveniently expressed as

$$p(\tau, \mu, \mu') = \frac{1 - g^2}{1 + g^2 - 2g\mu\mu'},$$

which was formulated by Henyey and Greenstein (1941). The parameter $g$ in Eq. (5.9) is called the asymmetry parameter, which is the average value of the cosine of the scattering angle (weighted according to the probability of scattering in the various directions). For symmetric scattering, which puts equal amounts of energy in the forward and backward directions, $g$ is zero. This is the case for optically small droplets, ice crystals, aerosol particles, as well as air molecules; this regime is called Rayleigh scattering. As more energy is scattered in the forward hemisphere, $g$ increases toward unity. Conversely as more energy is scattered in backward, $g$ tends toward $-1$. For 2-stream models, $g$ describes the partitioning of energy between the upper and lower hemispheres.

Another parameter defining the radiative properties of clouds is the optical thickness. Excluding the effects of water vapor, the formal definition of the optical thickness is

$$\tau = \int_0^z \int_0^\infty f(r)Q_e(x, n_\lambda)\pi r^2 dr dz,$$

where $Q_e(x, n_\lambda)$ is the extinction efficiency, and $f(r)$ is the spectral density of droplets of radius $r$. The first integral is over the cloud depth $z$. The second integral is over the cloud droplet radius $r$. Evaluation of Eq. (5.10) requires a knowledge of both the cloud droplet distribution and the behavior of $Q_e$. The extinction efficiency is defined as the ratio of the extinction to the cross-sectional areas of cloud droplets. For spherical cloud droplets, $Q_e$ can be evaluated from the Mie solution to the light scattering problem; this is an exact solution to the Maxwell equations. It is a function of particle radius through the size parameter $x = 2\pi r/\lambda$ and the refractive index of the particle $n_\lambda$.

Figure 5.4 illustrates the variation of the scattering efficiency factor for nonabsorbing water drops ($n_i = 0$) as calculated by Hansen and Travis (1974) using Mie theory. The extinction efficiency is the sum of the scattering and absorption efficiencies. The variation in the absorption efficiency as a function of $x$ also resembles Fig. 5.4. The efficiency was calculated assuming droplets were distributed in a gamma-type size distribution function that is determined by two parameters: $a = r_e$, the effective radius defined later in Eq. (5.15), and a coefficient of dispersion ($b$) about the effective radius $a$. The coefficient of dispersion is a measure of the droplet spectral width, with $b = 0$ being a monodisperse distribution and larger values representing broader spectra. Typical observed droplet-size distributions in cumuli exhibit broader spectra.
slightly less than 0.2. Remember that the parameter $x$ is a function of both the size of the droplets and the wavelength of electromagnetic radiation. Because the effective radius of most cloud droplet distributions is of the order of 10 $\mu$m, $x$ is typically greater than 10 for the visible spectrum. For large values of $x$, Fig. 5.4 illustrates (1) a series of regularly spaced broad maxima and minima called the interference structure, which oscillates about the approximate value of 2, and (2) irregular fine structure called the ripple structure. For broader droplet spectra, this fine structure is smeared out, but the feature of $Q_e$ approaching a limiting value of 2 for large $x$ remains.

Because for large values of $x$ (i.e. for shortwave radiation and typical cloud droplet-size distribution), $Q_e$ tends to be an almost constant value of 2, Eq. (5.10) can be written as

$$\tau = 2\pi \int_0^z \int_0^\infty f(r)r^2 dr dz.$$  \hspace{1cm} (5.11)

Thus Eq. (5.11) is only a function of the drop-size distribution and the depth of the cloud. To a good approximation we can write

$$\tau \simeq 2\pi N_c \bar{r}^2 h,$$  \hspace{1cm} (5.12)

where $N_c$ is the droplet concentration, $\bar{r}$ is the mean droplet radius, and $h$ is the
geometric thickness of a cloud layer. Using the notation in Chapter 4, we can express Eq. (5.12) in terms of the cloud droplet mixing ratio $r_c$ as

$$\tau \simeq 2\pi (3\rho_0/\pi \rho_w)^{2/3} h N_c^{1/3} r_c^{2/3},$$

(5.13)

where $\rho_0$ is the dry air density and $\rho_w$ is the density of water. The cloud droplet optical thickness is therefore a function of the cloud properties (liquid-water mixing ratio and cloud depth) as well as droplet concentration. The cloud droplet concentration, in turn, is largely determined by the aerosol content of the air mass or, in particular, the concentration of cloud condensation nuclei. For typical peak supersaturations in clouds, $N_c$ varies from as low as 10-50 cm$^{-3}$ for maritime clouds to 1000 cm$^{-3}$ for continental clouds. An important effect of changing the optical thickness of a cloud layer is that it changes the amount of reflected radiation and thereby alters the energy reaching the earth’s surface and the atmosphere below a cloud layer. Twomey suggests that if the CCN concentration in the cleaner parts of the atmosphere, such as oceanic regions, were raised to continental atmospheric values, about 10% more energy would be reflected to space by relatively thin cloud layers. He also points out that an increase in cloud reflectivity by 10% is of more consequence than a similar increase in global cloudiness. This is because, while an increase in cloudiness reduces the incoming solar energy flux, it also reduces the outgoing infrared radiation flux. Thus both cooling and heating effects occur when global cloudiness is increased. In contrast, an increase in cloud reflectance due to enhanced CCN concentrations does not appreciably affect infrared radiation but does reflect more incoming solar radiation, which results in a net cooling effect.

Stephens (1978a) has shown that another approximation to Eq. (5.10) is

$$\tau \simeq \frac{3}{2} W/r_e,$$

(5.14)

where $W$ is the liquid-water path (gm$^{-2}$) and $r_e$ is an effective radius defined as

$$r_e = \int_0^\infty f(r)r^3dr \Delta \int_0^\infty f(r)r^2dr.$$

(5.15)

The liquid-water path represents the integrated liquid-water through a cloud of depth $h$,

$$W = \int_0^h \rho_0 r_c dz.$$

(5.16)

Note that the effective radius is the ratio of the third moment of the droplet size distribution to the second moment. In other words it is proportional to the water content of a cloud divided by the total surface area of droplets. If one
imagines the same amount of water content of a cloud in a few big droplets, its effective radius will be larger than if the same water content were distributed on many small droplets. This is because, for the same water content, total surface area of many small droplets is greater than a few large droplets. Thus, effective radius conveniently describes two of the most important radiative cloud microphysical properties: volume and surface area.

It should be stressed that there is no instrument that directly measures effective radius. Nonetheless, effective radius has become an indispensable, if not abused, quantity in cloud remote sensing.

5.3.1. Absorption by Clouds

Absorption of solar radiation by clouds is generally quite small. Several investigators have estimated that precipitation-sized drops can appreciably increase cloud absorptance (Manton, 1980; Welch et al., 1980; Wiscombe et al., 1984). Because precipitation-sized drops form by collecting smaller cloud drops, they typically represent a miniscule percentage of the total drop concentration. Drizzle-sized drops have a concentration usually not exceeding \(100 \text{ m}^{-3}\), compared to cloud droplet concentrations of the order of \(10^5\) to \(10^9\) m\(^{-3}\). Thus precipitation-sized drops have little direct impact upon the reflectance of a cloud. On the other hand, because precipitation-sized drops can contribute significantly to the liquid-water content and, hence, to the liquid-water path, they can affect cloud absorption appreciably. Wiscombe et al. (1984) calculated that for a given liquid-water path, the absorptance is enhanced by 2-3% by the presence of large drops in a 4-km-deep cloud. Note, however, this is a small enhancement of a small amount of the total energy absorbed by clouds.

For a time it was thought that there was an appreciable difference, of as much as 40%, between the measured and the modeled cloud absorptance. This difference was called anomalous absorption (Cess et al., 1995; Ramanathan et al., 2003; Pilewskie and Valero, 1995). However, recent calculations using more sophisticated models and advanced measurement systems suggest that the difference between modeled and observed absorption is less than 10% which is within the range of instrument error (Ackerman et al., 2003; Sengupta and Ackerman, 2003).

5.3.2. Ice Clouds

The optical properties of ice clouds are complicated by the geometries of the ice particles, the uncertainties in ice crystal concentration, and their size spectra. We have seen in Chapter 4, that the habit of vapor-grown ice crystals varies with temperature and supersaturation with respect to ice. Moreover, as ice crystals grow by riming cloud droplets or aggregation, the geometrical and surface characteristics of the ice particles vary as do their optical properties. Important to any realistic assessment of the reflectance, transmittance, or absorptance of
an ice or mixed-phase cloud system is the concentration of ice particles and their size spectra, this is especially true of thin cirrus clouds. Unfortunately, as we have pointed out in Chapter 4, we do not have a reliable way of diagnosing or predicting ice crystal concentrations. Equation (5.12) suggests that errors in ice crystal concentrations can lead to large errors in estimating the shortwave optical thickness ($\tau$) for an ice cloud.

One further complication of ice clouds is that ice crystals are non-spherical and they are generally not randomly oriented in space. Instead, large ice crystals fall preferentially with their major axis oriented horizontal (Ono, 1969; Jayaweera and Mason, 1965; Platt, 1978), while smaller crystals tend to fall randomly oriented, depending on their Reynolds number (Sassen, 1980). Some small ice crystals with diameters of about 30 $\mu$m also exhibit preferred orientations but with large tilt angles (Klett, 1995).

Evidence of the complexity of the shortwave radiative properties of ice clouds can be seen from the variety of optical phenomena that are frequently observed. Features such as halos, sundogs, and pillars result from the interaction of shortwave radiation with ice crystals having a particular crystal habit, spatial orientation, size spectra, and a growth history that often includes a relatively turbulent-free, slowly rising environment (Hallett, 1987; Tricker, 1970; Greenler, 1980).

### 5.4. LONGWAVE RADIATIVE TRANSFER IN A CLOUDY ATMOSPHERE

In this discussion, we refer to longwave radiation as radiation emitted by the earth’s surface, or the atmosphere, having wavelengths greater than about 4 $\mu$m. The effect of clouds on longwave radiation is quite different from that of shortwave radiation. In the case of shortwave radiation, we find that cloud droplets are strong scatterers of incident radiation. Absorption of solar radiation by cloud droplets and ice crystals is small. By contrast, longwave radiation is strongly absorbed by cloud droplets in optically thick clouds. As much as 90% of incident longwave radiation can be absorbed in less than 50 m pathlengths in a cloud with high liquid-water content. Scattering of longwave radiation in clouds is secondary to absorption. Thus optically thick clouds are often considered to be blackbodies with respect to longwave radiation. Yamamoto et al. (1970) suggested that cumulonimbus clouds could be considered blackbodies after a pathlength of only 12 m. By contrast, the blackbody depth of thin cirrus ice clouds may be greater than several kilometers (Stephens, 1983), which is greater than the depths of those clouds. Thus, cirrus clouds, thin stratus, and many fogs do not behave as blackbodies over the infrared range.

We noted previously that, in a cloud-free atmosphere, little gaseous absorption takes place between 8 and 14 $\mu$m, a band which is commonly referred to as the “atmospheric window.” An exception is in the deep, maritime tropics where high values of low-level water vapor content can result in large amounts
of gaseous absorption. In an optically-thick cloudy atmosphere, on the other hand, there are no spectral regions where absorption of longwave radiation is small. Clouds therefore have a major impact on the amount of longwave radiation emitted to space. Thus clouds are extremely important to the earth’s climate.

The behavior of the extinction efficiency \( Q_e \) is quite different in the infrared region than it is over visible wavelengths for cloud particles. The extinction efficiency varies similarly to the behavior of the scattering efficiency shown in Fig. 5.4. Thus, for small values of \( x \), \( Q_e \) increases monotonically with \( x \). Moreover, at wavelengths corresponding to the peak in the spectral density of terrestrial radiative flux \((10 \mu m < \lambda < 20 \mu m)\), \( Q_e \) varies almost linearly with droplet radius for droplets with radii less than 20 \( \mu m \). Substitution of \( Q_e = kr \) into Eq. (5.10) yields

\[
\tau = \int_0^{\delta z} \int_0^\infty \pi kr^3 f(r) \, dr \, dz. \tag{5.17}
\]

Because the liquid-water content is

\[
\text{LWC} = \int_0^\infty \frac{3\pi}{\rho_\ell} r^3 f(r) \, dr, \tag{5.18}
\]

Eq. (5.18) shows that the optical thickness over the infrared range is principally a function of the liquid-water content of a cloud and is not strongly dependent upon the details of the cloud droplet spectrum. This greatly simplifies the parameterization of longwave radiative transfer through clouds.

The calculations by Wiscombe and Welch (1986), however, suggest that predictions of infrared cooling rates can be significantly affected by the presence of drizzle or raindrops near the tops of optically thick clouds. Estimated cooling rates for a cloud containing cloud droplets only, and for one containing precipitation, may differ by a factor of 4 in the topmost 50 m of a cloud. The differences between cooling rates for the two cloud types are reduced substantially at greater penetration distances in the cloud, although for an 8-km-deep cloud there is still nearly a factor of 2 difference in cooling rates between the two cloud types.

Paltridge and Platt (1976) noted that a commonly used approximation for estimating \( Q_e \) for complex-shaped crystals is to use the equivalent sphere approximation, Eq. (5.17). Mie theory can then be used to calculate the variation of \( Q_e \) as a function of \( x \). The result is similar to Fig. 5.4, for water drops, with differences due to the variation of refractive indices between water and ice.

Although the longwave radiative reflectance is small, Stephens (1980) calculated that it can have a significant impact on the flux profiles in the cloud and thus on the cloud-heating profiles. This is true when the upward flux from the earth’s surface is quite large. A longwave reflectance at the cloud base of only a few percent can thus significantly affect the upwelling fluxes at colder
cloud temperatures. This can substantially alter the strength of flux divergence and, hence, the rate of radiational cooling near the cloud top. This effect is most pronounced in the tropics.

A commonly used concept in longwave radiation diagnostic studies as well as parameterizations is the effective emittance concept. Cox (1976) determined cloud emittance values from measurements of broadband radiative flux profiles through clouds. The emittance can thus be defined

\[
\varepsilon(\uparrow) = \frac{F_B(\uparrow) - F_T(\uparrow)}{F_B(\uparrow) - \sigma T_T^4}
\]

for the upward irradiance, \( \varepsilon(\downarrow) = \frac{F_B(\downarrow) - F_T(\downarrow)}{\sigma T_B^4 - F_T(\downarrow)} \) for the downward irradiance.

\( F(\uparrow) \) and \( F(\downarrow) \) refer to the upward and downward measured infrared irradiances, respectively. The subscripts T and B refer to the top and bottom of the cloud layer, respectively, and \( \sigma \) is the Stefan-Boltzmann constant. The definition of effective emittance combines the effects of reflection, emission, and transmission by cloud droplets as well as gas molecules. The effective emittance is therefore not a scalar but a directionally dependent vector, since the emissivity is dependent upon the particular path the radiation takes through the atmosphere. As noted by Stephens (1980), when a cold cloud overlies a warm surface and reflects some longwave radiation, it can exhibit values of emittance considerably greater than unity.

5.5. RADIATIVE CHARACTERISTICS OF CLOUDS OF HORIZONTALLY FINITE EXTENT

Thus far we have considered only the radiative properties of horizontally infinite cloud layers. However, few cloud systems are horizontally homogeneous; a population of cumulus clouds is just one example. Even the tops of stratocumulus clouds are undulating, thus altering both the shortwave and longwave properties of those clouds.

Consideration of the finite geometries of clouds is quite complex. Looking out of the window at a few cumulus clouds, we observe the complicated shapes these clouds assume, sometimes identifying cloud shapes with animals or other familiar objects. Needless to say, describing such complicated shapes with mathematical functions or computer algorithms can be quite difficult. Often the cloud shapes are approximated as simple cubes (McKee and Cox, 1974, 1976; Davis et al., 1979a,b), while a few attempts have been made to simulate more complex shapes using superimposed sinusoidal functions (Takeuchi, 1986). But more recently the science of 3D cloud radiative transfer has made great advances as summarized in the book edited by Marshak and Davis (2005). Analytical modeling techniques such as the Fourier-Ricati approach of Gabriel et al. (1993) and the analytical-numerical methods of Evans (1998) have greatly advanced
our understanding of 3D radiative transfer as well as many recent observational programs. Still at the time of this writing the state-of-the-art has not advanced to the point where 3D cloud resolving models interact explicitly with 3D radiative transfer. On the mesoscale and larger, a stochastic approach to parameterizing the 3D radiative properties of clouds may be needed as suggested by Gabriel et al. (1993).

In general, it appears that the finite geometrical properties of clouds are more important to the bulk radiative properties than are variations in the cloud microstructure.

5.6. RADIATIVE INFLUENCES ON CLOUD PARTICLE GROWTH

Traditionally, cloud physicists have ignored the effects of radiation (Mason, 1971; Byers, 1965). It is generally argued that because the temperature difference between cloud droplets or ice crystals and their immediate surroundings is so small, nearly as much radiation is emitted from the cloud particle as is absorbed. This view is probably valid for a cloud particle that resides in the middle of an optically thick cloud. However, a cloud particle that resides at the top of a cloud layer or in an optically thin cloud such as a cirrus cloud essentially “sees” outer space, especially in the 8- to 12-μm spectral window. As a consequence, the surface temperature of the cloud particle will be cooler. As a result, the droplet or ice crystal will experience a higher supersaturation, or, in a subsaturated environment, a lesser subsaturation. This can be more readily seen by considering the rate of mass change due to vapor deposition or evaporation of an ice crystal or cloud droplet of mass $M$,

$$\frac{dM}{dt} = 4\pi CDf_1f_2[\rho_v(T_\infty) - \rho_s(T_s)], \quad (5.21)$$

where $C$ is the capacitance of an ice crystal as defined in Chapter 4. For a spherical droplet or ice crystal, $C = r$. The coefficient $D$ is the diffusivity of water vapor, $f_1$ is a factor that includes the accommodation coefficient for water molecules, $f_2$ is a ventilation function, $\rho_v(T_\infty)$ is the vapor density some distance from the particle surface, and $\rho_s(T_s)$ is the saturation vapor density with respect to ice or water at the surface temperature of the ice crystal. In order to estimate $T_s$, we normally assume that a cloud droplet or ice crystal is in thermal equilibrium,

$$L(\frac{dM}{dt}) + Q_r = 4\pi CKf_1f_2\Delta T, \quad (5.22)$$

where the first term on the left-hand side of Eq. (5.22) is the latent heat liberated in the growing cloud particle, $L$ is either the latent heat of sublimation or vaporization, and the second term on the left-hand side is the net radiative heating of the particle. The right-hand side represents the rate of heat diffusion away from the particle, where $K$ is the thermal diffusivity and $\Delta T$ is the temperature difference between a cloud particle and its environment. Roach
(a) (b) (c) (d) (e) (f)

FIGURE 5.5  Plots of \( dm/dr \) for selected times during the control run (TAP). For (a) 4.1 h, (b) 4.2 h, (c) 4.4 h, (d) 4.6 h, (e) 4.8 h, and (f) 5.0 h. The dashed line and solid line are for runs with (TAP) and with (TNR) radiation, respectively. Value or \( r_p \) are given in the plot. (From Harrington et al. (2000))

(1976) and Barkstrom (1978) considered the radiative effects on cloud droplet growth while Stephens (1983), Hallett (1987), and Wu et al. (2000) considered the radiative influences on ice crystal growth. Barkstrom showed that for optically thick clouds radiation can be important to droplet condensation for those droplets residing within 20 m of cloud top. For optically thin clouds, such as fogs and thin stratus, cloud droplets throughout the cloud may be affected by radiation. Harrington et al. (2000) examined the radiative effect on warm season Arctic stratus by using a trajectory ensemble model (TEM) driven by the flow fields produced by a two-dimensional eddy resolving model. They found that the longwave radiative (LW) effect reduced the time-scale for the onset of drizzle formation for up to 30 minutes. As shown in Fig. 5.5 droplet radiative cooling caused a broadening of the droplet spectrum such that more...
larger droplets formed, and smaller droplets less than 10 µm evaporated as they lost out in the competition for water vapor. The effect is most important for parcels of air residing near cloud top for 12 minutes or more.

Hartman and Harrington (2005a,b) examined radiative influences on the initiation of drizzle drops further by considering the competitive influences of LW and shortwave (SW) radiation using a TEM driven by a large eddy simulation model (LES). In contrast to LW radiation cooling, which is confined to layers of the cloud near cloud top, SW heating is effective throughout the depth of shallow clouds and acts to suppress droplet growth. However, because SW heating stabilizes the cloud layer, cloud parcel lifetimes are increased, resulting in longer cloud residence times. But because SW heating occurs throughout the cloud layer, droplet growth is retarded compared to cases with only LW radiation cooling. Hartman and Harrington (2005a) also examined the influence of droplet concentration on the cloud response to LW radiation cooling. They found that for low droplet concentrations LW cooling had little influence on the initiation of collection as collection was very active without LW radiation cooling. However, for droplet concentrations greater about 200 cm$^{-3}$ LW cooling accelerated the onset of collection. Hartman and Harrington (2005b) also examined the influence of solar zenith angle $\theta_0$. They found that at small $\theta_0$ solar heating dominates over LW cooling causing a suppression of collection for smaller droplet concentrations. For larger droplet concentrations LW cooling dominates over SW heating even at small $\theta_0$. At large $\theta_0$, SW heating does not alter the initiation of collection, thus LW cooling enhances collection for all droplet concentrations.

For the case of thin ice crystal clouds such as some cirrus clouds, Stephens also concluded that (1) because radiation can enhance (suppress) particle growth (evaporation), radiative cooling at cloud top and warming at cloud base tend to broaden and narrow the spectrum, respectively, and (2) the influence of radiation on the survival distance of falling ice particles is most significant in air having a relative humidity greater than 70%. At lower relative humidities evaporation is so strong that survival distances are altered little by radiation.

Wu et al. (2000) further examined the radiative influences on cirrus ice crystal growth using a 2D cloud resolving model. Both SW and LW radiation were considered. They found that with radiative feedback the cloud was optically thinner, allowing SW radiation to penetrate deeper into the cloud layer. Smaller crystals were little affected by radiational heating while larger crystals, having a larger radiation cross sectional area, experienced substantially reduced vapor deposition growth. Thus the size of the ice crystals was limited by radiational heating. It is interesting that, because the crystals were limited in size, they did not precipitate out of the cloud so much, and contributed to a longer-lived cirrus cloud.

It is clear that radiative effects can be important to cloud particle growth and evaporation.
5.7. AEROSOL EFFECTS ON THE RADIATIVE PROPERTIES OF CLOUDS

We have already seen that the concentration and the size spectra of cloud droplets have an important influence upon the shortwave radiative properties of clouds. The cloud droplet concentration, in turn, is largely a function of the concentration of cloud condensation nuclei activated in typical cloud supersaturations. The width of the droplet size spectra is to some extent also a consequence of the aerosol spectrum, although, as noted in Chapter 4, the width of the droplet spectrum is influenced by other cloud macroscopic parameters. We have also seen that the concentration and size spectra of ice crystals have a strong impact upon the radiative properties of clouds. Furthermore, the concentrations of aerosols active as ice nuclei are believed to play an important role in determining the ice crystal concentration, particularly for cold clouds such as altostratus and cirrus. Also, in those clouds having weak vertical motions, the size spectrum of ice crystals is largely determined by the competition for vapor among the ice crystals nucleated on ice nuclei. Thus the size spectra and chemical composition of aerosols have important controlling influences on the concentration of cloud droplets and ice crystals as well as on their size spectra, which, in turn, have important impacts upon the radiative properties of clouds.

In addition to affecting cloud radiative properties indirectly by influencing the cloud microstructure, aerosols can directly affect the radiative properties of clear as well as cloudy air. Assessment of the radiative effects of aerosols requires an estimate of the single scattering properties and, just as with cloud particles, a knowledge of the aerosol optical thickness. The optical thickness is determined by Eq. (5.10), which, like cloud droplets, requires a knowledge of the size distribution of the aerosol particles. Estimates of the extinction efficiency \( Q_e \) are complicated because aerosols are nonspherical, and, moreover, their variable chemical composition results in variability in their complex indices of refraction. Thus, the extinction efficiency varies with the source and life history of the aerosol. The life history is important because, as the aerosol ages, the particles coagulate with each other and form particles of mixed chemical composition. Furthermore, the extinction efficiency and the size spectra of the aerosol population change with relative humidity. At relative humidities greater than 70%, the hygroscopic aerosols take on water to become haze particles. As the relative humidity increases toward 100%, the aerosol particles swell in size and their complex indices of refraction change as the water-solution/particle mixture changes in relative amounts.

Computations of the radiative effects of natural dry aerosols suggest that polluted boundary layer air can result in shortwave radiative heating rates of the order of a few tenths of a degree to several degrees per hour (Braslau and Dave, 1975; Welch and Zdunkowski, 1976). Above the boundary layer, the aerosol shortwave radiative heating rates are much less. Even in the Saharan dust layer, heating rates are only of the order of 1-2 °C per day (Carlson and Benjamin,
1980; Ackerman and Cox, 1982). Nonetheless, such heating rates strengthen the overlying inversion, which further concentrates the pollutants in the lower troposphere. Absorption of solar radiation by aerosols can result in stabilization of a moist moderately stable layer and weaken convection and precipitation. This effect of aerosols has been termed the semi-direct effect (Grassl, 1975; Hansen et al., 1997). The reduction in cloud cover associated with this effect can alter the surface energy budget significantly. If the aerosol contains a large fraction of soot, such as the south Asian haze, then warming of the aerosol layer can desiccate stratocumulus cloud layers and alter the properties of the trade-wind cumulus layer (Ackerman et al., 2003). The influence of black carbon has a dominant effect on absorption of solar radiation within the atmosphere, which leads to lower surface temperatures (Ramanathan et al., 2001; Lohmann and Feichter, 2001), and reduces outgoing fluxes.

The impact of aerosols on infrared radiative transfer is usually less than it is for shortwave radiation. The effect of aerosols is greatest in the atmospheric window, where gaseous absorption of infrared radiation is least (Welch and Zdunkowski, 1976; Carlson and Benjamin, 1980; Ackerman et al., 1976). Before longwave radiative cooling effects can be detected, rather large concentrations of aerosols through a deep layer must be present. Ackerman and Cox (1982) could not detect any significant change in longwave fluxes due to dust over Saudi Arabia. Carlson and Benjamin (1980) calculated that the Saharan dust layer can affect longwave radiative fluxes if the dust layer is sufficiently deep. Several researchers have found that longwave radiative cooling can be appreciable in polluted boundary layer air (Welch and Zdunkowski, 1976; Saito, 1981). Andreyev and Ivlev (1980) investigated the radiative properties of various organic and inorganic natural aerosols. They found that organic aerosols are typically less than 0.5 µm in radius and affect infrared radiation little, but have a significant impact on shortwave radiative fluxes. Infrared radiative fluxes were mainly affected by the presence of large \( r > 0.15 \) µm mineral substances. Andreyev and Ivlev’s evaluations did not include exposure of the aerosols to increasing relative humidity. Some of the small inorganic aerosols may be activated as haze particles at higher relative humidities. As a result, once the small aerosols have swollen in size, they may alter longwave radiative fluxes appreciably. Welch and Zdunkowski calculated that net longwave radiative fluxes changed by more than 25% from a dry polluted boundary layer compared to a moist polluted boundary layer.

The effect of increasing humidity on the radiative properties of aerosols is most pronounced in the shortwave spectrum. The swelling of aerosol particles or activation of haze particles has an appreciable impact on local visibility (Kasten, 1969; Takeda et al., 1986) and on the global albedo (Zdunkowski and Liou, 1976). Zdunkowski and Liou also calculate that the swelling of aerosol particles in humid atmospheres can alter the local albedo by as much as 5% relative to a dry atmosphere.
Aerosols can modify the radiative properties of clouds when they are mixed into a cloud system. The most hygroscopic of the aerosol particles participate in the nucleation of cloud droplets. Large and ultra-giant aerosols (greater than 1 µm in radius) rapidly become wetted regardless of their chemical composition, and become engulfed in cloud droplets or raindrops as a result of hydrodynamic capture. No longer functioning as aerosols, they still influence the radiative properties of the cloud system. Smaller, submicrometer-sized aerosols, however, can remain outside of cloud droplets (interstitial). Those that are non-hygroscopic may remain dry with little change in size. Hygroscopic aerosols, while not being activated as cloud droplets, will swell in size in the slightly sub-saturated or supersaturated environment. Thus, the interstitial aerosol population will resemble a cloud-free haze population, with the exception of the removal of the largest, most hygroscopic components by nucleation scavenging and hydrodynamic capture of the particles greater than 1 µm. After a time, the remainder of the submicrometer particles may also be scavenged by cloud droplets. The major scavenging processes for submicrometer aerosols are Brownian diffusion and thermophoretic/diffusiophoretic scavenging. As noted in Chapter 4, Brownian diffusion is quite slow, and because thermophoresis predominates over diffusiophoresis, for a submicrometer-sized aerosol, phoretic scavenging does not enhance scavenging of submicrometer aerosols in a supersaturated cloud. In subsaturated regions of a cloud, however, evaporating cloud droplets can be very effective at scavenging submicrometer aerosols by phoretic processes. The droplet spectrum has to be broad enough to allow the largest droplets to survive evaporation in local subsaturated regions, however. This means that in the least dilute regions of convective storm updrafts, the time scale is short owing to intense updraft speeds, and because little evaporation occurs in the weakly mixed regions, many submicrometer particles will survive the ascent into the upper troposphere. At least for some time, submicrometer aerosol particles may remain interstitial and affect the radiative properties of the stratiform region of those clouds.

The presence of the ice phase also contributes to scavenging of interstitial aerosols. Slowly settling, branched snowflakes can be very effective removers of any remaining particles greater than 1 µm. Likewise, as ice crystals sublimate in sub-ice-saturated environments, they can be effective scavengers of submicrometer aerosols by phoretic processes. Furthermore, as we noted in Chapter 4, when ice crystals grow by vapor deposition, they do so at the expense of cloud droplets, causing their evaporation. Again, if the droplet spectrum is broad enough, the larger droplets may survive evaporation and contribute to the scavenging of submicrometer aerosols. Still, these processes are slow enough that the anvils or stratiform regions of deep convective systems, especially in polluted environments, contain a substantial interstitial aerosol population for some time. A significant interstitial aerosol population also exists in the tops of boundary layer stratocumuli and relatively young middle-tropospheric stratus clouds.
Assuming that the interstitial aerosol size spectrum is unchanged by the presence of cloud, Newiger and Bahnke (1981) calculated that aerosols can enhance absorption in a 4-km-thick horizontally uniform cloud by factors of 1.3 to 2.4.

The next question concerns how the scavenged aerosol particles affect the radiative properties of clouds. If the particles are soluble, they will dissolve once they have become embedded in water droplets for a time. Only in very small droplets will the solution be concentrated enough to affect the refractive index of the droplet. If the droplet totally evaporates, however, they become aerosol particles again (perhaps altered in size and chemical composition) and affect radiative transfer again. If the particles, however, are insoluble, as are graphitic carbon or soot, they can remain embedded in droplets and alter the radiative properties of the droplets. If the insoluble particles are greater than 1 µm in radius, they could be scavenged by hydrodynamic capture. As noted previously, submicrometer-sized insoluble particles can be scavenged by Brownian or phoretic scavenging processes, or they can coagulate with hygroscopic particles and be removed by nucleation scavenging. Chylek et al. (1984) calculated the radiative properties of soot particles embedded in droplets. Assuming that the soot particles were randomly distributed throughout the droplets, they showed that the absorption efficiency at short wavelengths of graphite carbon particles embedded in droplets is more than twice the efficiency of the same particles freely suspended in air, as long as the volume fraction of the particles in water is small. Figure 5.6 illustrates the calculated change in cloud reflectance and absorptance as a function of the volume fraction of graphite carbon for optically thick clouds having three different droplet size spectra. The cloud absorptance is substantially enhanced while the reflectance is reduced for soot volume fractions greater than $10^{-5}$. Whether this amount of graphitic carbon occurs in cloud droplets in nature is unknown at this time. Certainly in the case of the hypothesized nuclear winter scenario (see, e.g. Pittock et al., 1986) one would expect to find volume fractions of graphitic carbon in excess of those shown in Fig. 5.6.

What would be the effects of graphitic carbon particles attached to ice crystals? Again, little is known about the range of volume fractions of carbon that become attached to ice crystals. One would expect that graphitic carbon particles attached in sufficiently high numbers to ice crystals would decrease the reflectance of ice clouds substantially. There is certainly evidence that soot embedded in surface snow significantly affects the albedo of the snow surface (Warren, 1982).

5.8. PARAMETERIZATION OF RADIATIVE TRANSFER IN CLOUDS

5.8.1. Introduction

We have seen in previous sections that clouds have an important impact upon radiative transfer. Because divergence of radiative fluxes contributes
to heating/cooling in a cloud system, radiative transfer processes can alter the thermodynamic stability of a cloud system and thereby contribute to the dynamics of the system. Because of the complexity of other cloud processes and their computational demands, cloud dynamicists and mesoscale modelers must seek simplifications in the formulation of radiative processes in models. Much of the research has been aimed at formulating radiative transfer parameterization schemes suitable for use in general circulation models. General circulation models typically do not have enough vertical resolution to make realistic estimates of the liquid-water path, let alone the other cloud properties such as hydrometeor type and spectra, and the finite dimensions of clouds.

5.8.2. Parameterization of Shortwave and Longwave Radiation in Clouds

One general approach to reducing the complexities associated with the solution to (5.6) is to introduce the so-called “two-stream approximation.” In this approach the total radiation field is hemispherically-averaged, and represented by two streams: one in the upward direction (↑) and one in the downward direction (↓). The radiation intensity $I(\tau, \mu)$ is then integrated over the upward
and downward hemispheres to define the fluxes

\[ F_{\uparrow \downarrow} (\tau) = \int_{0}^{1} \mu I(\tau, \pm \mu) d\mu, \quad (5.23) \]

which, as Eqs (5.1) and (5.2) show, are the quantities needed to determine solar heating. Meador and Weaver (1980) showed that by assuming that \( I \) is dependent on \( \mu \), the hemispheric integral of Eq. (5.6) reduces to standard ordinary differential equations of the form

\[ \frac{dF_{\uparrow}}{d\tau} = \gamma_{1} F_{\uparrow} - \gamma_{2} F_{\downarrow} + (F_{0}/4)\omega_{0}\gamma_{3}e^{-\tau/\mu_{0}}, \quad (5.24) \]

\[ \frac{dF_{\downarrow}}{d\tau} = \gamma_{2} F_{\uparrow} - \gamma_{1} F_{\downarrow} + (F_{0}/4)\omega_{0}\gamma_{4}e^{-\tau/\mu_{0}}. \quad (5.25) \]

The \( \gamma_{i} \) values are determined by the approximations used and are independent of \( \tau \). Solutions to Eqs (5.25) and (5.26) can be obtained by standard techniques for specified boundary conditions. Meador and Weaver (1980) showed that several standard solution techniques, such as the Eddington approximation and the quadrature methods, could be transformed into the standard form of Eqs (5.25) and (5.26), with the appropriate specification of the \( \gamma_{i} \) values. An illustration of a simple layered two-stream model is given in Fig. 5.7, where Re\((n)\) and A\((n)\) represent the reflectances and absorptances of each layer, respectively.
A few cloud resolving models (i.e. Fu et al., 1995) have used four-stream approximations as developed by Liou et al. (1988), Fu and Liou (1992). Although computationally more demanding than the two-stream approximation it offers more accuracy in radiative heating estimations particularly at higher latitudes where solar zenith angles are large.

Application of the two-stream or four-stream approximations to clouds requires the determination of the cloud properties $\tau$, $\tilde{\omega}$, and $g$ in terms of modeled or specified microphysical and macrophysical variables. For spherical droplets, these properties can be obtained using Mie theory (see van de Hulst, 1957). If, however, the particle shape is complex, such as is the case for ice crystals or aerosols, solutions are not easily obtainable using Mie theory. In this case approximate solutions are necessary. For very large particles ($x = 2\pi r/\lambda \gg 1$), it has become customary to employ geometrical optics, in which the paths of individual rays traveling through a droplet or ice particle are traced. Rays passing through a particle and those not interacting with the particle are not allowed to interact or interfere with each other. Figure 5.8 illustrates an application of geometrical optics to the rainbow problem.

A somewhat less restrictive technique for obtaining the extinction and absorption efficiency factors is the so-called anomalous diffraction theory (van de Hulst, 1957; Ackerman and Stephens, 1987; Mitchell, 2000; Mitchell et al., 2006). Like geometrical optics, it is also valid for particles much larger than the wavelength of radiation ($x \gg 1$), and for which the index of refraction is $n \sim 1$, these are often referred to as soft particles. The anomalous diffraction approximation is based on the premise that the extinction of light is primarily a result of the interference between the rays that pass through a particle and those rays that are not influenced by a particle (see Fig. 5.9).

5.8.2.1. Cloud Optical Depth

Using Mie theory, or a suitable approximation to the interaction of radiation and droplets or ice particles, one can develop a parameterization of the cloud optical depth Eq. (5.10). Stephens (1978b) used eight different cloud droplet-size distributions to illustrate that the cloud optical depth, as calculated with Mie theory, is primarily a function of the cloud liquid-water path. He showed that $\tau$ could be approximated by Eq. (5.14). This is a useful approximation, because all one needs to do is calculate the liquid-water path, Eq. (5.16), and, given a climatologically derived effective radius, the optical thickness is easily determined.

A few cloud models using the two-stream approximation like Harrington (1997), Harrington et al. (1999) and Cotton et al. (2003) actually use the explicitly-resolved hydrometeor size distributions predicted with bin-microphysics models or diagnosed from two moment bulk schemes. This has the advantage that scattering and absorption by large raindrops and ice particles can be explicitly represented whereas use of the effective radius approximation masks the influence of these fewer but larger hydrometeors.
FIGURE 5.8 Path of light through a droplet can be determined by applying the laws of geometrical optics. Each time the beam strikes the surface, part of the light is reflected and part is refracted. Rays reflected directly from the surface are labeled rays of Class 1; those transmitted directly through the droplet are designated Class 2. The Class 3 rays emerge after one internal reflection; it is these that give rise to the primary rainbow. The secondary bow is made up of Class 4 rays, which have undergone two internal reflections. For rays of each class, only one factor determines the value of the scattering angle. That factor is the impact parameter: the displacement of the incident ray from an axis that passes through the center of the droplet. (From Nussenzveig (1977). Copyright © 1977 by Scientific American, Inc. All rights reserved)

FIGURE 5.9 Geometry of scattering by a large sphere with refractive index near 1. The solid ray passing through the sphere represents the anomalous diffraction theory (ADT), while the broken ray describes the ray path for the modified theory (MADT). (From Ackerman and Stephens (1987))
5.8.2.2. Single-Scattering Albedo

The single-scattering albedo also has been parameterized in terms of the effective radius. Based on geometrical optics, Liou (1980) developed the following parameterization of single-scattering albedo

\[ \tilde{\omega}_0 = 1 - 1.7k' r_e, \]  

(5.26)

where \( k' \) is the complex part of the index of refraction \( (n_\lambda = n_r - ik') \). Fouquart and Bonnel (1980) developed the expression

\[ \tilde{\omega}_0 = 1 + \exp(-2k' r_e), \]  

(5.27)

which is valid when liquid-water absorption is weak in the solar region. Using anomalous diffraction theory, van de Hulst (1957) derived a more general expression for \( \tilde{\omega}_0 \),

\[ \tilde{\omega}_0 = 1 - \frac{1}{2} \left( \frac{4}{3} \rho \tan \Gamma - \rho^2 \tan^2 \Gamma \right), \]  

(5.28)

where \( \rho = 2x(n_r - 1) \) and

\[ \Gamma = \arctan(k'/n_r - 1). \]  

Equation (5.28) is valid for small values of \( 4xk' \) and for \( Q_e = 2 \). Better estimates of \( \tilde{\omega}_0 \) can be obtained by using the tabulated values reported by Stephens et al. (1984). Stephens (1978b) showed that it is possible to remove the dependence of \( \tilde{\omega}_0 \) on \( r_e \) by empirically tuning \( \tilde{\omega}_0 \) as well as the integrated phase function parameter using accurate numerical solutions. Fouquart and Bonnel also used more accurate calculations to derive the spectrally-averaged, single-scattering albedo

\[ \tilde{\omega}_0 = 0.9989 - 0.0004 \exp(-0.15\tau). \]  

(5.29)

Here again, the single scattering albedo can be calculated from explicitly represented hydrometeor size distributions rather than using effective radius approximations.

5.8.2.3. Implementation in Cloud Resolving Models and Larger-Scale Models

In the past, computational efficiency has been gained in models by use of broadband approximations wherein the radiances are averaged across rather large wavelength bands (i.e. Stephens (1978b, 1984b), Harrington (1997)). However, many operational and research groups have engineered there radiation codes to include detailed line, two-stream models by using neural network techniques.
Fundamental Concepts and Parameterizations

Our discussion thus far has only dealt with a single homogeneous cloud layer with fixed values of \( \tau, \tilde{\omega}_0, \) and \( g. \) We now consider an atmosphere composed of a number of layers, each having different optical properties. The most commonly used approach is the so-called “adding” method. Grant and Hunt (1969a,b) considered an atmosphere composed of \( n \) homogeneous layers, each with their respective reflective (Re), transmissive (Tr), and absorptive (A) properties. For such an atmosphere, the reflectance \( \text{Re}(1, n + 1) \) represents the combined multiple reflectance contributed by all layers above the \((n + 1)\)th layer. It may be defined as

\[
\text{Re}(1, n + 1) = \text{Re}(n) + \frac{\text{Tr} \downarrow (n)\text{Tr} \uparrow (n)\text{Re}(1, n)}{1 - \text{Re}(1, n)\text{Re}(n)},
\]

(5.30)

where the reflection from a composite of all layers above the \((n + 1)\)th layer is obtained by adding the reflectance from two layers whose reflectance is \( \text{Re}(n) \) and \( \text{Re}(1, n). \)

The flux transmitted through the upper layer is represented by \( V \downarrow (n + 1/2) \) which may be computed as

\[
V \downarrow (n + 1/2) = \frac{\text{Tr} \downarrow (n)V \downarrow (n - 1/2)}{1 - \text{Re}(1, n)\text{Re}(n)}.
\]

(5.31)

Similarly, the flux transmitted from the lower layer \( V \uparrow (n + 1/2) \) is calculated as

\[
V \uparrow (n + 1/2) = \frac{\text{Re}(n)V \downarrow (n - 1/2)}{1 - \text{Re}(1, n)\text{Re}(n)}.
\]

(5.32)

The denominators in Eqs. (5.30)–(5.31) account for multiple reflections between layers. As noted by Stephens (1984a), this factor is especially large when the reflection between layers is large. This occurs when a dense cloud overlaps a bright surface such as another cloud or a snow-covered surface.

To close such a layered model, boundary conditions must be supplied. At the upper boundary, one can assume that the reflectance is zero and the downward-transmitted radiation corresponds to the flux coming into the atmosphere, or

\[
\text{Re}(1, 1) = 0,
V \downarrow (1/2) = F \downarrow (1).
\]

If the model top were placed at the tropopause, then \( F \downarrow (1) \) would correspond to the downward flux from stratospheric levels and above. At the lower boundary
the upward flux is given by

\[ F \uparrow (n + 1) = a_s F \downarrow (n + 1), \]

where \( a_s \) is the albedo of the earth’s surface. The value of \( a_s \) varies depending on whether the surface is dry land, vegetated, snow covered, or a sea surface. It also varies depending on solar elevation. Typical values of \( a_s \) for solar elevations of 45° are 7% for a water surface, 20-35% for dry grass lands, 30-40% for sand, and 80-85% for fresh snow.

This layered model illustrates the fact that the net radiative flux divergence is influenced by radiative transfer through all layers of the atmosphere above and below the level under consideration. To compute the transmittance and reflectance, we note that Eq. (5.3) can be integrated over all wavelengths shorter than 4 \( \mu \text{m} \) to give

\[ \text{Re} + \text{Tr} = 1 - A. \quad (5.33) \]

Thus, if a parameterization of absorptance and reflectance is available, one can compute the transmittance by using Eq. (5.33).

**5.8.2.5. Partial Cloudiness**

We have noted that the radiative properties of a population of finite clouds can differ substantially from that of a horizontally homogeneous cloud system. In the case of a region covered by partial cloudiness, the normal procedure is to weight the reflectance, transmittance, and absorptance calculated for a cloudy atmosphere by the cloud fractional coverage, and the corresponding clear-air properties by the clear-air coverage. This ignores contributions from radiation emitted from the sides of clouds and the interaction of radiation among neighboring clouds. As noted above, the full 3D radiative interaction among clouds has not yet been parameterized for use in cloud resolving or larger-scale models. A statistical approach is probably needed.

**5.8.3. Parameterization of Longwave Radiative Transfer in Clouds**

We have seen in Section 5.4 that clouds are effective absorbers of longwave radiation. A cloud of only modest liquid-water content of 0.2 g kg\(^{-1}\) may absorb up to 90% of the upwelling infrared radiant energy within a depth of only 50 m. By contrast, an equivalent penetration distance for shortwave radiation is at least 600 m for a cloud having the same liquid-water content. As a consequence, it is often assumed that optically thick clouds behave as blackbodies. Thus, all upwelling radiation is absorbed at the base of the cloud and is re-emitted in the upward and downward directions with a flux equal to \( \sigma T_b^4 \), where \( T_b \) is the cloud-base temperature. At the cloud top, the upward flux emitted by an assumed blackbody cloud will simply be \( \sigma T_T^4 \), where \( T_T \) is the temperature at
the cloud top. Many clouds, such as cirrus, stratus, and stratocumulus, as well as fogs, are not optically thick and, therefore, the blackbody approximation is a poor one. One must then seek alternate parameterizations of the longwave properties of such clouds.

A common approach used in many models until recently was to use the mixed emittance concept introduced by Herman and Goody (1976). However, it has become standard practice in most models to apply the two-stream or four-stream approximation methodology to longwave radiation as well.

5.9. SUMMARY

In this chapter we have reviewed the interaction between the cloud microphysical and macrophysical structure and radiative transfer processes. We have also presented some of the concepts and approaches to parameterizing radiative transfer through clouds. In Part II of this book we will examine the effects of radiative processes on the dynamics and precipitation processes in several different cloud systems.

REFERENCES


