Parameterization of the Atmospheric Surface Layer

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ABSTRACT

A parameterization of the constant flux surface layer is developed in order to provide boundary conditions for numerical models of the atmospheric boundary layer and moist convective layer. Algebraic expressions are found for the turbulence covariances in the surface layer under all stability conditions.

1. Introduction

Because the detailed structure of the flow near the ground apparently does not have a significant influence on the behavior of the overall troposphere, it is convenient to parameterize this surface layer when developing a numerical model of the atmospheric boundary layer. This means essentially that the lower boundary of the model is taken to coincide with the top of the surface layer. Second-order closure approximations for the turbulence covariances are now being introduced into numerical models of the atmospheric boundary layer (e.g., Deardorff, 1974). In these approximations the turbulence covariances are included as explicit dependent variables, and hence explicit boundary values of the covariances are required in order to determine the interior flow. Although there exist some field measurements of these covariances, the data are neither complete nor entirely consistent (see Monin and Yaglom, 1971).

Mellor (1973) presents a second-order closure model of the equilibrium surface layer which gives algebraic expressions for the turbulence covariances. However, the model yields unrealistic values for the turbulent normal stresses in very stable conditions. This arises apparently because the length scale of the turbulence is assumed to be independent of the stability of the layer. Lewellen and Teske (1973) overcome this problem by making the length scale constant when the layer is very stable. However, their model does not produce explicit algebraic expressions for the covariances; instead, a set of nonlinear ordinary differential equations must be solved.

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The purpose of the present work is to provide realistic estimates of the turbulence covariances at the lower boundary of a numerical model of the atmospheric boundary layer. To avoid excessive calculation in the determination of these boundary conditions, the surface layer model is contrived to yield explicit algebraic expressions for the covariances. This is done by following Mellor (1973), except for the introduction of a generalized turbulence time scale which yields reasonable values for the covariances under all stability conditions.

2. Turbulence covariances

We consider a surface layer in which the constant vertical fluxes of momentum, mass, heat and water vapor are represented by

$$\left.\begin{aligned} &\overline{u_1'u_3'}=-u_*^2, \quad \overline{\rho'u_3'}=B \\ &\overline{\theta'u_3'}=-H, \quad \overline{\rho_v'u_3'}=-W \end{aligned}\right\}, \tag{2.1}$$

where $\mathbf{u}$ is the velocity vector and ρ, θ and ρ_v are respectively the density, potential temperature and water vapor density of the air. An overbar denotes an average value while a prime denotes the deviation from the average. A Cartesian coordinate system (x_1,x_2,x_3) is used with x_3 increasing vertically upward and the velocity acts in the x_1 direction.

For an equilibrium surface layer the normalized mean profiles are universal functions of x_3/L where L is the Obukhov length. Thus it is found that (Monin and Yaglom 1971)

$$\left.\begin{aligned}\phi_m(\xi)&=\frac{kx_3}{u_*}\frac{\partial\bar{u}_1}{\partial x_3}\\ \phi_h(\xi)&=\frac{ku_*x_3}{H}\frac{\partial\bar{\theta}}{\partial x_3}=\frac{ku_*x_3}{W}\frac{\partial\bar{\rho}_v}{\partial x_3}\end{aligned}\right\}, \tag{2.2}$$

where $\xi=x_3/L$, $L=\bar{\rho}u_*^3/kgB$, $B/\bar{\rho}=(H/\bar{\theta})+[(R_v/R_a)-1]W/\bar{\rho}$; k is the von Kármán constant, g the gravitational acceleration, and R_v and R_a are the gas constants for water vapor and dry air, respectively. The functions ϕ_m and ϕ_h have been measured with some accuracy for moderate values of ξ; however, their behavior near the origin is still not completely clear (Dyer, 1974). Eqs. (2.2) can be integrated (Paulson, 1970) to yield relationships for u_*, H and W in terms of $\bar{u}_1$, $\bar{\theta}$ and $\bar{\rho}_v$ at some height. Hence the constant fluxes within the surface layer can be obtained from observations of the mean variables at the top of the layer (e.g., Deardoff, 1968; Clarke, 1970).

A second-order closure model of the equilibrium surface layer is now developed in order to predict all the turbulence covariances in terms of u_*, H and W. This is achieved by following Mellor (1973) and assuming that the surface layer is in complete local equilibrium with the local rate of production of turbulent energy precisely balancing the rate of dissipation. Thus terms describing molecular and turbulent transport are neglected in the covariance equations. Terms involving the covariance of the pressure fluctuation with the gradient of another fluctuating quantity represent the tendency of turbulence to approach a state of isotropy and they are modeled in a simple linear fashion (Rotta, 1951). The dissipation of turbulence occurs at scales small enough for its behavior to be isotropic, while the rate of dissipation is controlled by the large-scale motions. These terms are taken to be proportional to the variance of the dissipating quantity (Mellor, 1973). Because the mean wind acts in the x_1 direction, we have the symmetry condition

$$\overline{\rho' u_2'}=\overline{\theta' u_2'}=\overline{\rho_v' u_2'}=\overline{u_i' u_2'}=0,\quad i\neq 2. \tag{2.3}$$

Under the assumption that the transport mechanisms for heat and water vapor are similar (Monin and Yaglom, 1971), it follows that

$$\left.\begin{aligned}\overline{\rho'^2}/B^2=\overline{\theta'^2}/H^2=\overline{\rho_v'^2}/W^2=\overline{\rho_v'\theta'}/HW\\ -\overline{\rho' u_1'}/B=\overline{\theta' u_1'}/H=\overline{\rho_v' u_1'}/W\end{aligned}\right\}. \tag{2.4}$$

Hence the covariance equations reduce to

$$\left.\begin{aligned}-2u_*^2\partial\bar{u}_1/\partial x_3+(b_1/T)(\overline{u_1'^2}-\tfrac{1}{3}\overline{q^2})+\tfrac{1}{3}(a_1/T)\overline{q^2}&=0\\ (b_1/T)(\overline{u_2'^2}-\tfrac{1}{3}\overline{q^2})+\tfrac{1}{3}(a_1/T)\overline{q^2}&=0\\ 2Bg/\bar{\rho}+(b_1/T)(\overline{u_3'^2}-\tfrac{1}{3}\overline{q^2})+\tfrac{1}{3}(a_1/T)\overline{q^2}&=0\\ \overline{u_3'^2}\partial\bar{u}_1/\partial x_3+\overline{\rho' u_1'}g/\bar{\rho}-(b_1/T)u_*^2&=0\\ 2B\partial\bar{\rho}/\partial x_3+(a_2/T)\overline{\rho'^2}&=0\\ -u_*^2\partial\bar{\rho}/\partial x_3+B\partial\bar{u}_1/\partial x_3+(b_2/T)\overline{\rho' u_1'}&=0\\ \overline{u_3'^2}\partial\bar{\rho}/\partial x_3+\overline{\rho'^2}g/\bar{\rho}+(b_2/T)B&=0\end{aligned}\right\}, \tag{2.5}$$

where $\overline{q^2}=\overline{u_i'^2}$; T is a characteristic time scale for the turbulence; and a_1, a_2, b_1 and b_2 are constants.

In the absence of any density stratification, we expect the time scale T to be given by the mean strain rate of the flow. This may be expressed in terms of the positive invariant

$$1/T^2\propto(\partial\bar{u}_i/\partial x_j)^2.$$

Now, since buoyancy fluctuations ought to affect the time scale when the fluid is stratified, we generalize this to obtain

$$1/T^2=(\partial\bar{u}_i/\partial x_j)^2\psi^2(\eta), \tag{2.6}$$

where $\eta=(-\overline{\rho' u_3'}g)/(\bar{\rho}\overline{u_i' u_j'}\partial\bar{u}_i/\partial x_j)$ and $\psi^2(0)=1$. The parameter η is a flux Richardson number and it is the ratio of the rate of suppression of turbulent energy by buoyancy to the rate of production of turbulent energy by the working of the turbulent stresses against the mean strain rate; thus we expect the dimensionless function ψ^2 to decrease monotonically with η. The specification (2.6) for T ought to be applicable to more general flows than the present equilibrium surface layer.

The algebraic expressions (2.5) can be solved for the turbulence covariances. However, in neutral conditions, the equations for the Reynolds stresses form a linear homogeneous set and a nontrivial solution exists only if

$$a_1=2b_1/(2+3b_1^2). \tag{2.7}$$

Observations (see Table 1) show that $u_*^2/\overline{q^2}$ is equal to 0.16 in neutral conditions so we take

$$a_1=0.32. \tag{2.8}$$

It follows from (2.7) and (2.8) that

$$b_1=1.69. \tag{2.9}$$

TABLE 1. Comparison of predicted Reynolds stresses with observations in neutral conditions.

	$u_*^2/\overline{q^2}$	$\overline{u_1'^2}/\overline{q^2}$	$\overline{u_2'^2}/\overline{q^2}$	$\overline{u_3'^2}/\overline{q^2}$
Klebanoff (1955)	0.16	0.57	0.28	0.15
So and Mellor (1972)	0.16	0.53	0.27	0.20
Champagne *et al.* (1970)	0.17	0.48	0.28	0.24
Eqs. (2.13) and (2.14)	0.16	0.46	0.27	0.27

The constant b_2 is selected so that the ratio ϕ_h/ϕ_m equals 0.74 in neutral conditions, as observed by Businger *et al.* (1970); thus

$$b_2=1.25. \tag{2.10}$$

Finally, a_2 is found by ensuring that the limiting Richardson number $(\xi\phi_h/\phi_m^2)$, above which turbulence is not sustained, agrees with the observations of Businger *et al.* (1971). We therefore set

$$a_2=0.78. \tag{2.11}$$

Now, since the eight equations (2.5)–(2.6) contain nine variables the specification is completed by choosing ϕ_m to correspond essentially to the observations of Businger *et al.* (1971); in particular, we take

$$\phi_m(\xi)=\begin{cases}1+4.7\xi, & \xi\geqslant 0\\ (1-15\xi)^{-\frac{1}{4}}, & -0.5<\xi<0\\ 0.47(-\xi)^{-\frac{1}{3}}, & \xi\leqslant -0.5\end{cases} \tag{2.12}$$

The asymptotic behavior of ϕ_m as $\xi\to-\infty$ corresponds to the "free convection" similarity law (Monin and Yaglom, 1971). Although Businger *et al.* (1971) do not find this self-similar behavior in ϕ_m, their data are restricted to about $\xi>-2$ and measurements by Carl *et al.* (1973) for $\xi\gtrsim-150$ suggest that similarity is approached asymptotically. Eq. (2.12) varies from the Businger *et al.* result by less than 12% for $\xi\geqslant-2.5$.

Eqs. (2.4)–(2.12) can now be used to determine the turbulence covariances in the surface layer under any stability condition. We find that

$$\left.\begin{aligned}\overline{u_1'^2}&=(0.27q_0^2+1.18/\psi)u_*^2\\ \overline{u_2'^2}&=0.27q_0^2u_*^2\\ \overline{u_3'^2}&=(0.27q_0^2-1.18\eta/\psi)u_*^2\\ \overline{\rho'^2}/B^2=\overline{\theta'^2}/H^2=\overline{\rho_v'^2}/W^2&=\overline{\rho_v'\theta'}/WH\\ &=2.56\phi/\psi u_*^2\\ -\overline{\rho'u_1'}/B=\overline{\theta'u_1'}/H=\overline{\rho_v'u_1'}/W&=0.80(1+\phi)/\psi\end{aligned}\right\}, \tag{2.13}$$

where

$$\left.\begin{aligned}\eta&=\xi/\phi_m(\xi)\\ \phi&=\phi_h/\phi_m=0.74(1-2.18\eta)/(1-2.86\eta)\\ \psi^2&=(1-3.21\eta)(1-2.18\eta)/(1-2.86\eta)\\ q_0^2&=6.25(1-\eta)/\psi\end{aligned}\right\}. \tag{2.14}$$

The normalized height ξ is given by (2.2) with $k=0.35$ and the profile function $\phi_m(\xi)$ is obtained from (2.12).

Turbulence can be also sustained by an unstable stratification in the absence of any mechanical generation; that is in pure free convection. The asymptotic form of T as $\eta\to-\infty$ is found from (2.6) by replacing the turbulent fluxes in η by their equivalent eddy diffusivity representations. Hence we take

$$\overline{\rho'u_3'}=-K_h\partial\bar{\rho}/\partial x_3 \quad\text{and}\quad \overline{u'_1u_3'}=-K_m\partial\bar{u}_1/\partial x_3,$$

where K_h and K_m are the turbulent diffusivities of mass and momentum. As a result we have

$$\eta=-\frac{K_hg}{\bar{\rho}}\frac{\partial\bar{\rho}}{\partial x_3}\left[K_m\left(\frac{\partial\bar{u}_1}{\partial x_3}\right)^2\right]^{-1}.$$

Therefore, from (3.7) and (2.14c),

$$1/T^2=2.45(K_h/K_m)(g/\bar{\rho})(\partial\bar{\rho}/\partial x_3)\ \text{as}\ \eta\to-\infty. \tag{2.15}$$

Also from (2.14b) we have

$$K_m/K_h=\phi_h/\phi_m=0.56\ \text{as}\ \eta\to-\infty. \tag{2.16}$$

Thus Eqs. (2.15) and (2.16) yield

$$1/T^2=4.34(g/\bar{\rho})(\partial\bar{\rho}/\partial x_3)\ \text{as}\ \eta\to-\infty.$$

The turbulence covariances can now be found directly from (2.5) with $\bar{u}_1=u_*=0$.

3. Comparison with observations

The turbulent Reynolds stresses predicted by (2.13) and (2.14) in neutral conditions are shown in Table 1 where they are compared with the observations of Klebanoff (1955), So and Mellor (1972) and Champagne *et al.* (1970). As in the model of Mellor (1973), theory implies that $\overline{u_2'^2}=\overline{u_3'^2}$ whereas observations show $\overline{u_2'^2}$ to be somewhat greater than $\overline{u_3'^2}$. The theory predicts too large a transfer of energy from the x_1 direction into the x_3 direction. However, considering that only one constant (a_1) is adjusted to obtain these results, the predictions appear satisfactory.

The Reynolds stresses, normalized with respect to u_*^2, are found from (2.13) and (2.14) to approach constant values in stable conditions, in at least qualitative agreement with the limited data (Monin and Yaglom, 1971). Wyngaard *et al.* (1971) find that the normalized density covariances are approximately constant in stable conditions. Although their data are very scattered, their results compare favorably with the predictions of (2.13) and (2.14); namely,

$$\left.\begin{aligned}(\overline{\rho'^2}u_*^2/B^2)^{\frac{1}{2}}=1.97,\quad -\overline{\rho'u_1'}/B=2.42\\ -\overline{\rho'u_1'}u_*^2/\overline{u_3'^2}B=1.47\quad\text{when}\quad \eta=0.21\end{aligned}\right\}.$$

In unstable conditions, the model yields values which correspond to free convection similarity theory (Monin and Yaglom, 1971). In particular, it is found that

$$\overline{u_3'^2}/u_*^2\sim 2.68(-\xi)^{\frac{2}{3}}\quad\text{and}\quad \overline{\rho'^2}u_*^2/B^2\sim 0.63(-\xi)^{-\frac{2}{3}}$$
$$\text{as}\quad \xi\to-\infty,$$

which may be compared with the observed results of Wyngaard *et al.* (1971) in unstable conditions:

$$\overline{u_3'^2}/u_*^2=3.6(-\xi)^{\frac{2}{3}} \quad \text{and} \quad \overline{\rho'^2}u_*^2/B^2=0.90(-\xi)^{-\frac{2}{3}}.$$

The present model is seen to specify consistently the turbulence covariances in a constant flux layer under all stability conditions. The predicted values are in reasonable agreement with existing observations.

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