ELSEVIER

Atmospheric Research 47–48 (1998) 529–553

ATMOSPHERIC RESEARCH

A critique of one- and two-dimensional models of boundary layer clouds with a binned representations of drop microphysics

Bjorn Stevens [a,*], William R. Cotton [b], Graham Feingold [c]

[a] Advanced Study Program, National Center for Atmospheric Research, Boulder, CO 80304, USA
[b] Department of Atmospheric Science, Colorado State University, Fort Collins, CO, USA
[c] Environmental Technology Laboratory, NOAA / CIRA, Boulder, CO, USA

Abstract

A variety of models of boundary layer turbulence are increasingly being coupled to binned representation of cloud-drop spectra for the purpose of studying cloudy boundary layers and aerosol-cloud-drop interactions. A critique of one dimensional models shows that extant methods of coupling bin-microphysical models to the turbulence model omit important terms involving covariances of perturbation quantities. Errors incurred from the omission of these terms are significant: cloud-drop activation is not properly represented, and the initial stages of precipitation growth may be retarded in horizontally inhomogeneous boundary layers. If the promise of one-dimensional models is ever to be realized, some means of accurately representing these terms must be developed. Because large-scale models often have grid-spacings much larger than typical cloud-eddy-scale circulations, attempts to couple them to more detailed microphysical representations will also suffer from the problems discussed in the context of one-dimensional models. This should provide added motivation for improving the representation of microphysical processes on the sub-grid scale. An analysis of two-dimensional eddy-resolving models closed on the basis of energy inertial range arguments illustrates that the lack of an energy cascade in two dimensions results in increasingly vigorous circulations as the model's grid-mesh is refined. Two-dimensional model simulations can display significant variability to vanishingly small differences in the initial conditions, and drizzle rates vary by as much as 30% when total water concentrations are increased by only 1%. Such sensitivities make it difficult to constrain the models given the degree of uncertainty that characterizes measurements of state variables in clouds. Despite some inconsistencies in the formulation of their closure terms, two-dimensional models are at least able to qualitatively represent the interaction of the large-eddies and microphysical processes in boundary layer flows. For this reason they appear to be useful, not as surrogates for reality, but

* Corresponding author. Tel.: +1-303-497-8932; e-mail: bstevens@ucar.edu

PII S0169-8095(98)00059-3

rather as a means for further refining hypotheses about the physical system thereby increasing the chances that such hypotheses can be tested observationally.

Keywords: Cloud-drop; Bin-microphysical models; Energy cascade

1. Introduction

In recent years a number of papers have appeared which make use of or describe a one-dimensional turbulence-closure model coupled to a binned representation of the distribution of liquid water. For instance, this type of model has been used to postulate that boundary layers may collapse in the absence of sufficient aerosol concentrations (Ackerman et al., 1993), to further evaluate Baker and Charlson's hypothesis that CCN concentrations are effectively bi-stable with respect to aerosol production rates (Ackerman et al., 1994), and to study the possibility that clouds absorb substantially more solar radiation than previously thought (Ackerman and Toon, 1996). Early papers, in which this type of model was used to postulate or examine physical hypotheses, insufficiently described the model; hence, results had to be taken largely on faith (e.g., Ackerman et al., 1993, 1994). Recent descriptions of the models (Ackerman et al., 1995; Bott et al., 1996) indicate that important terms in the equations for the time evolution of the horizontally averaged droplet spectrum are ignored. As we discuss in Section 2, the omission of these terms may explain why, in some respects, the models behave unphysically, particularly near cloud base.

Another approach, has been to couple three-dimensional turbulence models to binned representations of cloud microphysics (Kogan et al., 1994; Kogan et al., 1995; Stevens et al., 1996a). This approach addresses many of the deficiencies inherent in current formulations of one-dimensional models, but it, too, has difficulties. For one, a representation of mixing across cloud interfacial boundaries is largely a spurious process in Eulerian models with microphysics forced by a grid-box averaged supersaturation field (Stevens et al., 1996b). In addition, representing the condensation process in the Eulerian spatial domain requires vertical grid-spacings of no more than a few tens of meters (e.g., Clark, 1974; Stevens et al., 1996a). To represent the turbulence properly, at least the beginnings of an inertial range should be resolved. Thus there exist both dynamical and microphysical constraints on the maximum size of the grid mesh. These constraints coupled with a binned representation of the microphysics, make the cost of three-dimensional simulations prohibitive. Only relatively few short-time scale phenomena may be examined, and then only in domains of a relatively limited extent [$\mathscr{O}$(km)]. As a result the formulation is not well suited to steady-state studies of aerosol cloud-drop interactions; to achieve steady results (over which meaningful averages may be taken) integrations need to be longer than average aerosol lifetimes, which may be on the order of days.

Two-dimensional models are often an appealing middle ground, because they explicitly resolve 'turbulent-like' eddies, yet they can be orders of magnitude less computationally intensive than their three dimensional counterparts. Moreover, recent comparisons between two- and three-dimensional integrations suggest that two-dimensional models of convective flow represent important processes reasonably (Moeng et al., 1996), and thus, for some problems, may be an adequate framework within which to model turbulent flows forced by buoyancy. Nonetheless, two-dimensional turbulence does not possess an energy-inertial range, and is therefore fundamentally different from its three-dimensional counterpart. This energy-inertial range provides the theoretical basis for many of the closures used in two- and three-dimensional models. However, because the closure is based on arguments intrinsic only to three-dimensional flows (Lilly, 1967; Mason, 1994), we wonder about the impact of its use in a two-dimensional model.

Critical discussions of methods (such as the one we attempt here) are not common, perhaps because most studies focus on an exploration of physical ideas, or a particular response in the model, in which case such a detailed and critical discussion of the models behavior can be distracting. However, as modelers we often understand the shortcomings of our models better than others, and thus we have a responsibility to inform the wider community of the flaws or weaknesses which underlie our work. This is not to say that critical discussions are nonexistent. Telford (1996), for instance, criticized the treatment of turbulence and its interaction with microphysical processes in numerical models of clouds, but his discussion lacks rigor and currency (it does not discuss any cloud model written within the past decade). Stevens et al. (1996a) used observations to assess the merits of their model qualitatively, and performed numerical experiments designed to isolate sources of numerical error, but discussions such as theirs can often be tedious. Ackerman et al. (1995) and Bott et al. (1996) discuss some limitations of their models and attempt detailed comparisons of their results with observations. However, and as we shall discuss, such comparisons can often be misleading.

This paper adds nothing to our knowledge of actual clouds, instead it provides a critique of some numerical models which are used to advance our understanding. Specifically we take the opportunity provided by this special issue on cloud modeling to discuss some important shortcomings (or limitations) in current formulations of one- and two-dimensional turbulence models coupled to binned representations of the cloud microphysics. We argue that there are fundamental oversights in the formulation of *existing* one-dimensional turbulence cloud models with bin microphysics. These oversights (while not insurmountable) call into question the ability of existent one-dimensional models to accurately represent processes which depend strongly on an ability to correctly model the cloud microphysical interactions. We further argue that two-dimensional models can provide a qualitatively reasonable framework for the study of some processes that depend on cloud microphysical structure, but that inconsistencies in their formulation lead to a significant sensitivity to grid-mesh size; moreover, it is argued that the source of this grid-mesh-size sensitivity problem may be responsible for a distortion of certain physical processes.

This paper does not address the limitations of three-dimensional models. This is not because there are none. Indeed, in previous papers we have examined the fidelity of three-dimensional models in some detail. In so doing we showed that three-dimensional models suffer from the cloud-edge problem common to all existent models which represent the cloud on a fixed grid (Stevens et al., 1996b), as well as resolution problems (Stevens, 1996a). However, available computational resources have not changed sufficiently over the past couple of years to warrant further study of these issues.

Nor does this paper attempt to compare the results from one- two- and three-dimensional models. Such a comparison would be interesting, and perhaps useful. However, having previously explored the errors associated with three dimensional models, our purpose here is to consider the implications of uncertainties in the formulation and closure of existent one- and two-dimensional models—specifically we wish to highlight limitations associated with the lack of (or improper) representation of the sub-grid scale processes in these models.

2. Bin-microphysical models coupled to Eulerian dynamical models

Coupling a bin-microphysical model to any numerical model that does not represent all scales of motion requires some sort of closure (i.e., a representation of the unknown quantities in terms of known quantities) for nonlinear terms. To illustrate this point we must consider the derivation of the system of equations used by limited resolution numerical models.

2.1. Development of equations describing the evolution of the drop-spectrum in an Eulerian spatial domain

Let $n(m,x_i,t)\mathrm{d}m$ denote the number of drops in the vanishingly small interval $(m,m+\mathrm{d}m)$. Here $x_i=(x_1,\ x_2,\ x_3)=(x,y,z)$ denotes the spatial, and t the temporal, dependence. Knowledge of the processes affecting this drop-distribution function enables us to write a partial differential equation (PDE) describing its evolution. To a degree of approximation sufficient for our discussion, this equation can be written as follows:

$$
\begin{aligned}
\frac{\partial n(m,x_i,t)}{\partial t} = & -\frac{\partial\left[u_j(x_i,t)\,n(m,x_i,t)\right]}{\partial x_j} - \frac{\partial\left[w_t(m)\,n(m,x_i,t)\right]}{\partial x_3} \\
& -\frac{\partial\left[m^{1/3}n(m,x_i,t)\mathscr{S}(x_i,t)\right]}{\partial m} - n(m,x_i,t)\int_0^\infty n(q,x_i,t) \\
& \times\mathscr{K}(m,q)\mathrm{d}q + \frac{1}{2}\int_0^m n(m-q,x_i,t)\,n(q,x_i,t) \\
& \times\mathscr{K}(m-q,q)\mathrm{d}q. \qquad (1)
\end{aligned}
$$

The first term on the right-hand side is the advection term, describing the transport of substance by an incompressible fluid through an Eulerian reference frame. The second term is the sedimentation term, describing the gravitational settling of drops of different sizes. The third term describes the growth of drops by vapor diffusion with $\mathscr{S}(x_i,t)$ being a thermodynamic function roughly proportional to the supersaturation. The last two terms represent stochastic collection and describe the rearrangement of drop mass by random collisions. The kernel, $\mathscr{K}$, describes the joint probability of such collisions occurring and leading to coalescence. Here $\mathscr{K}$ is not dependent on the flow field, although current research suggests that it may be.

Eq. (1) is continuous in space, time and drop mass, and for the cases of interest allows no known analytical solutions, and thus it must be solved numerically. As a PDE, it has an infinite number of degrees of freedom, which must be reduced to a manageable number. A common approach is to approximate the above system by a discrete analog which is truncated at some scale. This truncation is based on a decomposition of spatially dependent variables into mean and perturbation quantities. For instance, n is represented by some average value $\bar{n}$ and its deviation n' from that average (which we call a perturbation). Often this decomposition is assumed to obey Reynolds averaging rules. This need not be the case, but because this assumption simplifies the formulation, it will be used throughout the remainder of our report.

This decomposition of variables into a mean and deviations from the mean is a basic step in deriving most numerical models. In one-dimensional turbulence-closure models, $\bar{n}$ represents an ensemble average; whereas in two- or three-dimensional models $\bar{n}$ represents n averaged over a grid volume. Because we know of no models that attempt to integrate equations for n' or any of its moments, we consider only the equation that describes the evolution of $\bar{n}$. Replacing n with $\bar{n}+n'$ in Eq. (1), and averaging results in

$$\begin{aligned}\frac{\partial \bar{n}(m,x_i,t)}{\partial t} = &-\frac{\partial}{\partial x_j}\left[\overline{u_j}(x_i,t)\,\bar{n}(m,x_i,t) + \overline{n'(m,x_i,t)\,u_j'(x_i,t)}\right] \\ &-\frac{\partial\left[w_t(m)\,\bar{n}(m,x_i,t)\right]}{\partial x_3} - \frac{\partial}{\partial m}\Big[m^{1/3}\bar{n}(m,x_i,t)\,\mathscr{S}(x_i,t) \\ &+m^{1/3}\overline{n'(m,x_i,t)\,\mathscr{S}'(x_i,t)}\Big] + \frac{1}{2}\int_0^m\Big[\bar{n}(m-q,x_i,t)\,\bar{n}(q,x_i,t) \\ &+\overline{n'(m-q,x_i,t)\,n'(q,x_i,t)}\Big]\mathscr{K}(m-q,q)\,\mathrm{d}q - \int_0^\infty\Big[\bar{n}(m,x_i,t) \\ &\times\bar{n}(q,x_i,t) + \overline{n'(m,x_i,t)\,n'(q,x_i,t)}\Big]\mathscr{K}(m,q)\,\mathrm{d}q. \end{aligned} \tag{2}$$

Eq. (2) is similar to Eq. (1) except for the presence of four new terms. These terms all involve correlations between perturbation quantities and henceforth are referred to as sub-filter scale or correlations of sub-filter scale terms. The first such term describes the correlation of perturbation motions with perturbations in the drop spectrum. Covariances between conserved scalars and the velocity field are widely discussed, and often modeled diffusively by assuming the existence of an eddy diffusivity which linearly relates the covariance to the mean gradient of the scalar. However, many state variables are not conserved in the presence of phase changes, as a result (and as we discuss below) modeling their evolution using such an approach can often be problematic. The second new term can be thought of as a stochastic condensation term; it comes from the quadratic nonlinearity in the drop-growth equation and describes the effect of correlations between drop size and supersaturation on the rate of change of the mean-size distribution. It, too, has been widely discussed (Belyaev, 1967; Stepanov, 1976; Clark and Hall, 1979; Cooper, 1989). The last two terms arise from the nonlinear term in the collection integral, and, depending on the nature of the deviations from the mean in the drop distribution, they may lead to either a relative narrowing or broadening of the spectrum. Typically, all but the first additional sub-filter term are ignored in numerical models. Our purpose is in part, to review the merits of this often unspoken-assumption.

All of the terms in question involve deviations from a mean quantity, where the mean is often an implicit quantity dependent on both the model that is being integrated, and the spatial resolution of that model. Hence any discussion of the importance of the various subgrid terms must be model (and sometimes resolution) dependent. First we discuss these issues in the context of a one-dimensional turbulence closure model which foregoes a description of the n' terms. Later we discuss the impact of these terms within the framework of a two-dimensional model of turbulence.

2.2. Coupling to one-dimensional turbulence-closure models

In a one-dimensional turbulence-closure model, the overbar represents an ensemble average. If, as is commonly done, we (i) assume that the ensemble statistics are horizontally homogeneous, [1] (ii) represent the nonlinear term arising from advection by a down-gradient diffusion term (with some diffusivity K), and (iii) neglect the remaining terms involving n', we arrive at the following equation for the evolution of $\bar{n}(m,z,t)$:

$$
\begin{aligned}
\frac{\partial \bar{n}(m,z,t)}{\partial t} = {} & \frac{\partial}{\partial z}\left[K\frac{\partial \bar{n}(m,z,t)}{\partial z}\right] - \frac{\partial\left[w_t(m)\,\bar{n}(m,z,t)\right]}{\partial z} \\
& - \frac{\partial\left[m^{1/3}\bar{n}(m,z,t)\,\mathscr{S}(z,t)\right]}{\partial m} - \int_0^\infty \bar{n}(m,z,t)\,\bar{n}(q,z,t) \\
& \times \mathscr{K}(m,q)\,\mathrm{d}q + \frac{1}{2}\int_0^m \bar{n}(m-q,z,t)\,\bar{n}(q,z,t) \\
& \times \mathscr{K}(m-q,q)\,\mathrm{d}q. \qquad (3)
\end{aligned}
$$

This is essentially the same equation solved by models that couple a one-dimensional

[1] Homogeneity in the statistics causes the horizontal derivatives of expected values to vanish. It does not mean that state variables are assumed to be horizontally homogeneous.

turbulence-closure model to a binned representation of the microphysics (Ackerman et al., 1995; Bott et al., 1996). It has several problems.

2.2.1. Forcing the microphysics with mean supersaturations

Stevens et al. (1996a) use a three-dimensional model to show that although a layer may be subsaturated in the mean near (or just above) cloud base, it is locally supersaturated in regions of updrafts. In this region the aerosol responds to the local production of excess vapor and activate (i.e., grow in a regime characterized by unstable steady states). The maximum supersaturation in this region, which is determined by the strength of the forcing and the characteristics of the aerosol spectrum (Twomey, 1959), determines the fraction of the soluble aerosol population that become cloud drops. Because the initial character of the droplet distribution has such a strong influence on the subsequent microphysical evolution of the cloud, any model that attempts a binned-representation of the cloud-drop spectrum must be able to represent this process. Failure to do so calls into question the subsequent evolution of the microphysics *irrespective* of the degree of sophistication of the microphysical model.

However, in a one-dimensional model, the above-described process is completely dependent on the sub-filter scale correlations. In other words, the correlations between perturbation supersaturations with perturbation aerosol concentrations, i.e., the $\overline{n'(m,t)\mathscr{S}'(t)}$ term in Eq. (2), describe how cloud condensation nuclei become activated near cloud base in updrafts. *This is precisely the term that is neglected in all published accounts of attempts to link a one-dimensional turbulence-closure model to a bin-microphysical model*. The models of both Ackerman et al. (1995) and Bott et al. (1996) exemplify the errors associated with the assumption that $\overline{n'(m,t)\mathscr{S}'(t)} \approx 0$. For instance, because the mean supersaturation is increasing in cloud (and doesn't become positive until about midway through the cloud), activation in such models occurs in the upper half of the cloud and at a relatively low supersaturation. [2] Cloud-top drops are diffused (by the parameterized eddy diffusivity) toward the base. Moreover, because there is no activation at the cloud base, haze drops must be included in the cloud-drop concentration profiles in order to yield reasonable looking results (see Figs. 4 and 9 of Ackerman et al., 1995 and Fig. 15 of Bott et al., 1996). Note that this is not necessarily a reflection of errors in the mean supersaturation field, it is merely a statement of the fact that the layer averaged supersaturation has little (if anything) to do with the microphysical processes in the cloud.

2.2.2. Down-gradient diffusion of non-conserved variables

Traditionally one-dimensional turbulence-closure cloud models have used a bulk microphysical assumption that enables both cloud water and the buoyancy flux to be

---

[2] Ackerman et al. (1995) attribute their unrealistic supersaturation profiles to the assumption of one-dimensionality. On the contrary, if proper closures can be found, one-dimensional models are perfectly suited to the incorporation of horizontal inhomogeneity. Hence the problem is not one of horizontal-homogeneity or one-dimensionality, rather it follows from the fact that important forcing terms are ignored.

solved for diagnostically as a function of conserved variables. By a conserved variable we mean one (such as ψ) that, in the absence of molecular diffusion, satisfies the following relation:

$$\frac{\partial\psi}{\partial t}+\frac{\partial\left(u_j\psi\right)}{\partial x_j}=0. \tag{4}$$

Using the decomposition discussed above enables us to write the following equation for $\overline{\psi}$:

$$\frac{\partial\overline{\psi}}{\partial t}+\frac{\partial\left(\overline{u_j}\overline{\psi}\right)}{\partial x_j}=-\frac{\partial\left(\overline{u_j'\psi'}\right)}{\partial x_j}. \tag{5}$$

When a variable is not conserved, [i.e., there is some forcing on the right-hand side of Eq. (4), which we denote by $\mathscr{F}(\psi)$] Eq. (5) takes the form:

$$\frac{\partial\overline{\psi}}{\partial t}+\frac{\partial\left(\overline{u_j}\overline{\psi}\right)}{\partial x_j}=\mathscr{F}\left(\overline{\psi}\right)-\frac{\partial\left(\overline{u_j'\psi'}\right)}{\partial x_j}+\left[\overline{\mathscr{F}(\psi)}-\mathscr{F}\left(\overline{\psi}\right)\right], \tag{6}$$

where the last two terms on the right-hand side may be thought of as a single term representing the effects of any sub-filter correlations associated with a nonlinear source term. In essence, this is a compact way of writing Eq. (2).

If one models this equation by parameterizing the collective impact of subgrid scale effects using a single term $\mathscr{P}_\psi(\overline{\phi_i})$, where $\overline{\phi_i}$ represents the state variables of the model so that

$$\frac{\partial\overline{\psi}}{\partial t}+\frac{\partial\left(\overline{u_j}\overline{\psi}\right)}{\partial x_j}=\mathscr{F}\left(\overline{\psi}\right)+\mathscr{P}_\psi\left(\overline{\phi_i}\right), \tag{7}$$

then there must be some justification for the use of $\mathscr{P}_\psi$, particularly in regard to its ability to represent the nonlinear terms in the forcing function $\mathscr{F}$.

Simply representing covariances between velocities and scalars in terms of local scalar gradients and eddy-diffusivities [i.e., setting $\overline{u_j'\psi'}=-K\frac{\partial\overline{\psi}}{\partial x_j}$ in Eq. (6)] and neglecting the forcing terms in $\mathscr{P}_\psi$ can be problematic. Apart from the fact that in buoyancy-driven flows mixing is well known to be locally upgradient, such a procedure may unrealistically project the forcing terms onto the resolved scales. It may be a spurious source for the mean supersaturation profiles generated by one-dimensional models.

Consider the equation for the available vapor $\eta=q_\mathrm{v}-q_\mathrm{s}(\theta,p)$ at a fixed pressure level:

$$\mathrm{d}\eta=\mathrm{d}q_\mathrm{v}-\mathrm{d}q_\mathrm{s}=\mathrm{d}q_\mathrm{v}-\alpha\,\mathrm{d}\theta, \tag{8}$$

where α is a thermodynamic parameter which depends weakly on temperature and pressure (but for our purposes may be considered constant), q_v is the vapor mixing ratio and q_s is the saturation mixing ratio. The assumption that covariances between velocities and scalars are proportional to local scalar gradients implies that for one-dimensional models

$$\frac{\mathrm{d}\eta}{\mathrm{d}t} = \frac{\partial}{\partial z}\left[K\left(\frac{\partial q_v}{\partial z} - \alpha\frac{\partial\theta}{\partial z}\right)\right] - \eta/\tau. \tag{9}$$

Here we have neglected mean vertical motion and assumed we are in a cloud layer and replaced $\mathrm{d}\theta/\mathrm{d}t$ and $\mathrm{d}q_v/\mathrm{d}t$ in Eq. (8) with their forcing terms, the effect of condensation (which arises in both the θ and q_v equations) has been represented by a relaxation term η/τ where τ is a timescale roughly proportional to the integral radius of the liquid water distribution. Hence, in the absence of gradients in the eddy diffusivity, concavity in $\theta(z)$ and convexity in $q_v(z)$ is a source of η. Because at cloud top, the profiles of θ and q_v are typically concave and convex respectively (convex and concave at cloud base) this formulation leads to a production of excess vapor near cloud top, and a destruction of it near cloud base (see Bott et al., 1996 for a more intuitive discussion of this issue). This response is difficult to interpret on physical grounds (it seems to be a simple artifact of representing covariances as linearly proportional to local gradients and neglecting the effect of sources acting on the scale of the turbulence), yet it is in accord with (and may well explain) the behavior of the one-dimensional models.

One might argue that the use of K-theory to represent the effect of turbulence on non-conserved variables is reasonable if (a) diffused quantities are tracers of air motions, and (b) the sources acting on non-conserved variables act on timescales much longer than the mixing timescales. The failure of large drops to satisfy the first criterion motivates some to neglect the effects of small-scale turbulent motions on large drops (e.g., Krueger, 1985). While this may be reasonable for two- or three-dimensional models (a point we examine further in Section 3) it presents a dilemma for one-dimensional models in which *all* the turbulence is represented by the sub-filter terms. Bott et al. (1996), checked the second criterion and find that it is supported by their model. Apart from the fact that their model makes the assumption which is to be tested, their conclusion must be questioned on two grounds. First, their estimate of the condensation timescale is probably too large. This is because the condensation timescale varies inversely with the integral radius of the drop distribution, which in their one-dimensional model is underpredicted (due to the problems in adequately representing cloud base activation). Second, their estimate of the turbulent timescale uses the large-eddy convective velocity scale, and a local length scale. If the large-eddy velocity scale is to be used, then the appropriate lengthscale (for a well mixed boundary layer) is the boundary layer depth. This incongruency in choice of velocity and lengthscales leads to at least an order of magnitude underestimate in the turbulence timescale.

2.2.3. *Neglect of horizontal variability in solving collection*

Chen and Cotton (1987) point out that local variations in liquid water may be important in deriving the layer averaged drizzle rates. Martin et al. (1995), show that large, local, horizontal deviations in the cloud-drop spectral properties do occur in stratocumulus cloud, particularly when cumulus penetrate a stratus deck from below. Furthermore, they speculate that these deviations may be important in promoting drizzle growth. To estimate the magnitude of this term we used the model described by Stevens et al. (1996a), with the collection process represented as in Feingold et al. (1996a,b). We use a simulation of strongly-precipitating stratocumulus, [i.e., the three-dimensional ASTEX (Atlantic Stratocumulus Transition Experiment) simulations with a small mean wind in Stevens (1996)], and compare the rate at which large drops are produced by the model, with the rate large drops are produced if we only use the layer-mean drop distributions in the collection calculations.

Fig. 1 shows the liquid-water mixing ratio at the level at which the collection comparisons were made. This snapshot is taken about 80 min after the turbulence has fully developed and from a level (midway through the cloud layer) where the production of layer-averaged drizzle drops is large. The inhomogeneity in this field arises from the fact that cloud base is irregular and the boundary layer is not perfectly well mixed (partly in response to the drizzle that has been falling during the course of the simulations). In this discussion, we consider the simulated liquid-water mixing ratios to be representative of the inhomogeneities typically found in precipitating stratocumulus. In generating Fig. 2, we used the model-produced cloud-drop spectrum (which had already developed a significant layer-averaged drizzle mode) as the initial condition for the collection calculations. Collection was carried out for 10 min. Fig. 2 and Fig. 3 compare calculations that predict the evolution of $\bar{n}(m,t)$ as a function of $n(m,x,y,t)$ with those that predict the evolution of $\bar{n}(m,t)$ as a function of $\bar{n}(m,t)$. In other words,

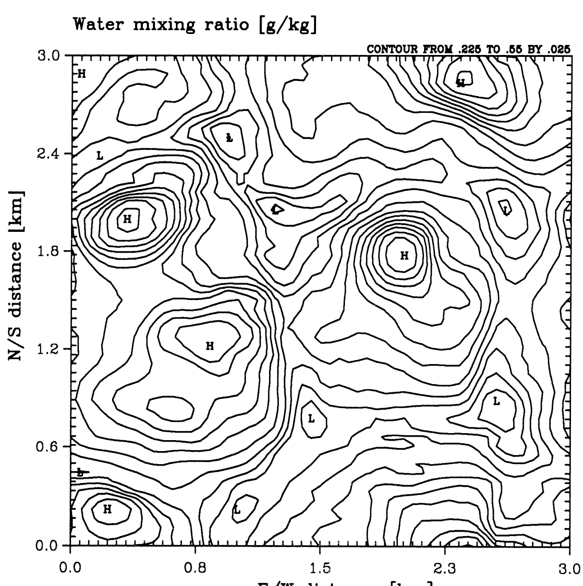

Fig. 1. Plan view of liquid water mixing ratio in the middle of a drizzling stratocumulus layer. Contour interval and range given at top of plot.

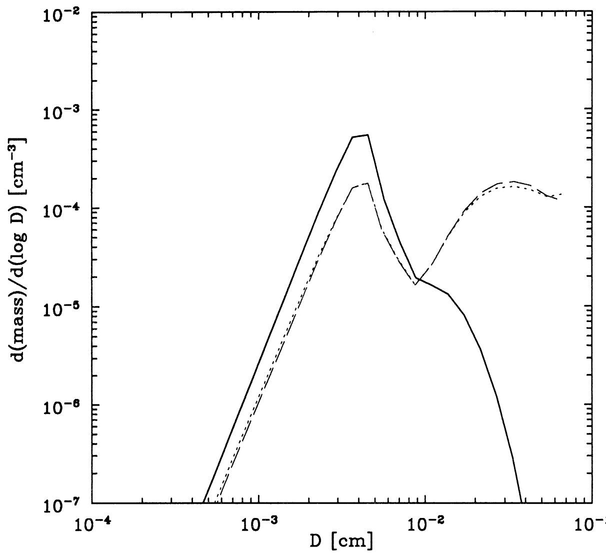

Fig. 2. Comparison of drop mass spectral distributions after 10 min of collection. Initial conditions (solid), mean drop distribution after 10 min with collection using only layer averaged spectrum (dashed), mean drop distribution after 10 min when collection is done independently at every point in the two-dimensional cross-section of Fig. 1 (dotted).

the latter calculations neglect the contribution of covariances in the drop spectrum (the $\overline{n'n'}$ terms in the collection integrals) to the evolution of $\bar{n}(m,t)$.

Surprisingly, deviations from horizontal homogeneity had little effect on the subsequent evolution of the pre-existing layer-averaged drizzle mode. This was not the case when we initialized the calculations with a narrow (perhaps unrealistically so) lognormal spectrum ($\sigma_g = 1.2$, $N = 25$ cm^{-3}) and allowed the collection to proceed for 20 min. Here, departures from horizontal homogeneity were very important for the initial growth of the drizzle mode (see Fig. 3) from a very narrow distribution. However, this sensitivity is mitigated for broader distributions.

These calculations are not definitive. It may well be that the horizontal inhomogeneity produced by the LES model over or under estimates the inhomogeneity in real clouds. Moreover, the magnitude of the sensitivity clearly depends on the breadth of the initial drop distribution. Nonetheless, our results suggest that even if one-dimensional models were able to represent the cloud base activation process correctly, their ability to evaluate hypotheses dependent on the relationship between sub-cloud CCN concentrations and precipitation formation would remain questionable in the absence of a further quantification of this term. Another interesting implication of these results is that large scale models which attempt to relate precipitation production to grid-scale values probably need to account for some measure of sub-grid variability.

2.3. Coupling to eddy-resolving models

Two-dimensional models have the advantage of being able to reasonably represent the boundary-layer scale overturning associated with the large eddies (Moeng et al.,

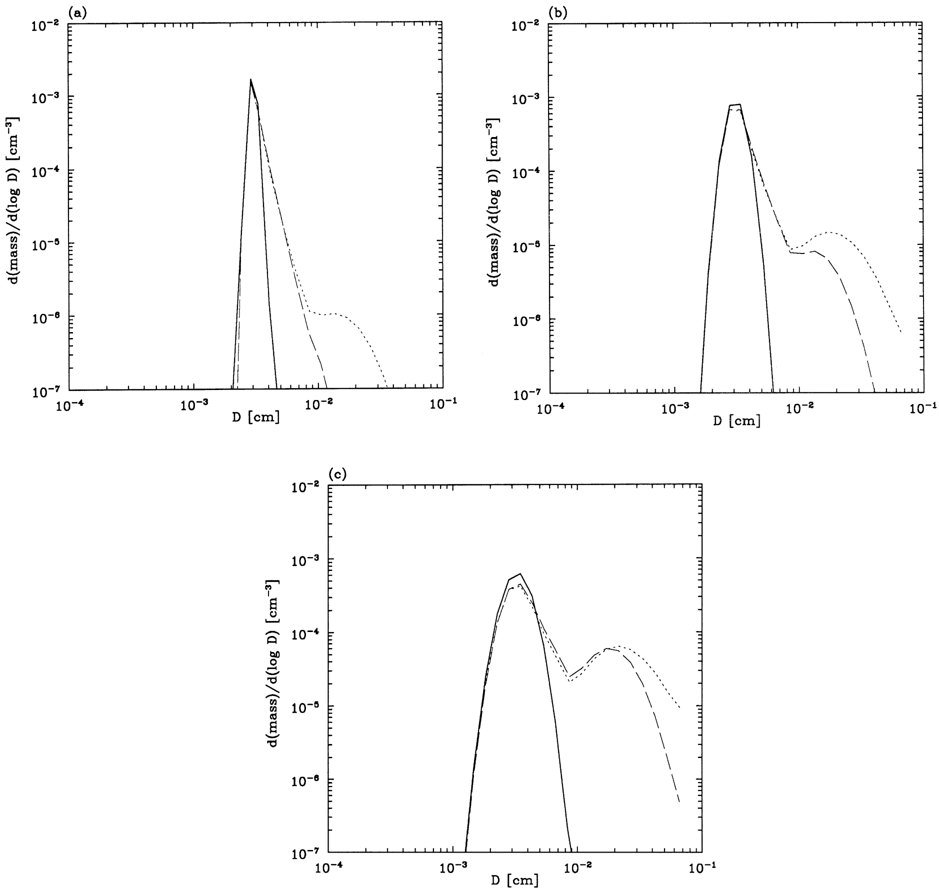

Fig. 3. Same as Fig. 2 but without the presence of a drizzle mode in initial conditions. Breadth parameter for initial drop distribution: (a) $\sigma_g = 1.2$; (b) $\sigma_g = 1.6$; (c) $\sigma_g = 2.0$.

1996), and for this reason, they are often called eddy-resolving models (ERMs), terminology which we have adopted here. Nonetheless, as in any model that does not represent the full range of scales evident in the physical system, two-dimensional models are vulnerable to the closure problem discussed above. However, interpretations of the terms that need to be closed differs. Now the sub-filter correlation terms should be interpreted as the effect of departures from the local-volume averages taken over the two-dimensional grid. In other words, they represent the effects of three-dimensionality and small (grid-scale) motions. The equation describing the evolution of the drop-distribution function in the two-dimensional model typically is:

$$\frac{\partial \bar{n}(m,x,z,t)}{\partial t} = \frac{\partial}{\partial z}\left[K_z \frac{\partial \bar{n}(m,x,z,t)}{\partial z}\right] + \frac{\partial}{\partial x}\left[K_x \frac{\partial \bar{n}(m,x,z,t)}{\partial x}\right]$$

$$
\begin{aligned}
&-\frac{\partial}{\partial m}\left[m^{1/3}\bar{n}(m,x,z,t)\mathscr{S}(x,z,t)\right]-\frac{\partial\left[w_t(m)\bar{n}(m,x,z,t)\right]}{\partial z}\\
&-\frac{\partial\bar{u}(x,z,t)\bar{n}(m,x,z,t)}{\partial x}-\frac{\partial\overline{w}(x,z,t)\bar{n}(m,x,z,t)}{\partial z}\\
&-\int_0^\infty\bar{n}(m,x,z,t)\bar{n}(q,x,z,t)\mathscr{K}(m,q)\mathrm{d}q
\end{aligned}
\tag{10}
$$

$$
2q+\frac{1}{2}\int_0^m\bar{n}(m-q,x,z,t)\bar{n}(q,x,z,t)\mathscr{K}(m-q,q)\mathrm{d}q. \tag{11}
$$

Apart from the additional functional dependence on x, the inclusion of advection terms associated with the resolved wind field, and the addition of an eddy-diffusivity term in the horizontal direction, the form of this equation is identical to the form of Eq. (3); but, because a large portion of what was left unrepresented in the one-dimensional model is now represented through the additional functional dependence on x, its content is significantly different. Instead of neglecting or parameterizing the effects of all the turbulence, we must neglect or parameterize only effects that are smaller than the filter scale. Whether or not interactions on this scale can be neglected remains uncertain. However, because the energetic scales of the boundary layer are well resolved there seems to be considerably more justification for neglecting subgrid scale forcing terms and representing subgrid scale transport terms diffusively. Support for this point of view is provided by the two-dimensional simulations themselves, investigations of which show that the rudimentary aspects of the cloud microphysical structure are well reproduced (Feingold et al., 1994; Feingold et al., 1996a,b; Feingold et al., 1997).

3. Sensitivity of two-dimensional model results to numerical formulation

Notwithstanding that the ERM model can represent ‘turbulent-like’ eddies, which enables it to represent much of the energy-containing range of the turbulent boundary layer, it is worthwhile to attempt to quantify the sensitivity of the model to a variety of the uncertainties inherent in the formulation of the ERM. Chief among these uncertainties is the nature of the two-dimensional flow, and its relationship to the grid-mesh size. Because it is not obvious how to examine the effect of neglecting the subfilter terms directly we approached the issue indirectly by examining how different representations of subfilter terms and different choices of model gridspacing impact the solutions. We also examined the sensitivity of the ERM to a variety of other uncertainties.

3.1. Model description and initialization

The ERM uses a finite-difference approximation in the anelastic equation for a rotating fluid. Slab symmetry is assumed in the meridional direction. Total water mixing ratio, q_{T}, and liquid-water potential temperature, θ_1 are the primary thermodynamic variables. Boundary conditions are cyclic in the zonal direction, and a rigid lid is used for the upper and lower boundaries. Fifty additional variables, which represent the zeroth and first moments of the liquid-water-mass distribution in 25 size categories, are also prognosticated. Drops are activated by using a diagnostic (fixed in time) CCN concentration, and the processes of condensation and evaporation, drop sedimentation, and stochastic collection are explicitly represented. The dynamics of the aerosol are not treated physically because sources or sinks of aerosol are not included. This is a severe limitation of the model. We have attempted a more realistic representation of these processes (e.g., Feingold et al., 1996b), however such a representation is computationally intensive. Because we did not believe the extra costs were warranted (given that the focus of the current discussion is on issues relating to the interactions of the drop spectrum with the boundary-layer thermodynamic structure and turbulence on timescales shorter than the aerosol equilibration time) the more detailed representation of the aerosol was not included. More details regarding the model are provided by Stevens et al. (1996a), and Feingold et al. (1996a,b).

All integrations are initialized using the following piecewise-linear initial conditions (where height z is in meters, θ_1 is in Kelvin, and q_T is in g kg^{-1}):

$$(\theta_1, q_T) = \begin{cases} (288.0+\delta\theta_1, 10.2) & z \le 662.5 \\ (288.0+\delta\theta_1, 10.2) \quad +(0.110, -0.0220)\cdot(z-662.5) & 662.5 < z \le 687.5 \\ (288.0, 10.2) \quad +(0.110, -0.0220)\cdot(z-662.5) & 687.5 < z \le 712.5 \\ (293.5, 9.1) \quad +(0.006, -0.0028)\cdot(z-712.5) & \text{otherwise} \end{cases} \tag{12}$$

A pseudo-random perturbation, $\delta\theta_1 \in (-0.1,\ 0.1)$, is applied at each grid-point and forced to satisfy $\int_X (\delta\theta_1) \mathrm{d}x = 0$ at every level. A large-scale pressure gradient is assumed to be in balance with geostrophic winds of -2 and 10 ms^{-1} in the zonal and meridional directions respectively.

The model is forced by cloud top cooling the magnitude of which we derive from a simple parameterization of the effects of long-wave radiation, such that in each column of the model the radiative flux is a function only of liquid-water path, LWP, i.e.,

$$F_{\mathrm{rad}}(z) = F_0 e^{-\alpha[\mathrm{LWP}(z)]}, \text{ where } \mathrm{LWP}(z) = \int_z^{z_{\mathrm{top}}} \rho_0 q_1 \mathrm{d}z. \tag{13}$$

Here F_0 is the maximum rate at which energy can be extracted, and α is a parameter which regulates the depth of the cloud layer over which this extraction takes place, ρ_0 is the basic state density, q_1 is the liquid-water mixing ratio and z_{top} is the top of the model. In accord with the third GCSS (GEWEX cloud systems studies) case-study we initially choose $\alpha = 130$ m^2 kg^{-1} and $F_0 = 74$ Wm^{-2}, which leads to cooling rates of about 8 K h^{-1} being confined to the top 25 m of cloud. This is an admittedly crude parameterization of a rather complex process; however, intercomparisons with other models (see Moeng et al., 1996) and experiments of our own with more complicated two-stream schemes have led us to conclude that such a scheme adequately represents the leading order effects of the longwave radiative forcing at cloud top. For the CCN distribution, we assume a narrow log-normal spectrum $(D_g,\ \sigma_g) = (0.2\ \mu\mathrm{m},\ 1.5)$. This distribution tends to make drop and CCN concentrations commensurate, since the former are readily accessible, given the range of supersaturations typically produced. For all experiments, CCN concentrations were fixed at approximately 100 cm^{-3}. The large-scale divergence was fixed at 5×10^{-6} s^{-1} (and added as a source term to the budget equations for scalar quantities). Most of the studies in this paper were carried out with a grid-mesh discretization of $(\Delta x, \Delta z, \delta t) = (50$ m, 25 m, 2 s). A progressively stretched grid was used above 800 m, with a grid-stretch ratio of 10% to the model top at 1500 m. A Rayleigh-friction-damping layer was applied in the upper 400 m (7 layers) of the domain with a damping time-scale of 60 s.

3.2. Baseline tests

To evaluate the significance of the model response to a change in a specific parameter, we conducted three 8-h baseline simulations. The only difference among the integrations was the choice of the initial random seed (i.e., the sequence of random perturbations to the initial temperature field was changed). The integrations generate a tremendous amount of information, and it is difficult to come up with simple scalar measures that characterize the behavior of the integrations. We have chosen to look at five parameters, none of which is necessarily independent of another, but each of which should contribute to a better understanding of the gross properties of each integration. These parameters are: (i) $\mathscr{L}$ the domain-averaged value of LWP; (ii) $\mathscr{D}$, the domain-averaged drizzle flux across the lower boundary in units of Wm^{-2}; (iii) Z_I, the cloud-top height, which we define to be the uppermost point at which half of the model columns are cloudy; (iv) $w_2 = \langle \overline{w'w'} \rangle^{1/2}$, which is the square-root of the boundary-layer averaged value of the vertical-velocity variance and, thus, a measure of the turbulence activity; (v) $\Delta\Theta_1$, a measure of the stability of the sub-cloud layer defined by the difference between $\overline{\theta_1}$ averaged over all completely cloudy layers and $\overline{\theta_1}$ at the third model level. The statistics of $\mathscr{L}$ and $\mathscr{D}$ are relatively steady after the 4 h, and therefore are averaged over the period from 4 to 8 h. The other variables are averaged only over the last 3 h.

Results are given in Table 1. Immediately apparent is the significant variability among the integrations. The standard deviations of $\mathscr{D}$ and $\Delta\Theta_1$ are 20% of the mean, and the variations in $\mathscr{L}$, w_2, and Z_I, are also appreciable. The nonlinear nature of the drizzle production contributes to the variability in most fields. Variances in the non-precipitating integrations are smaller (see Table 2), but perhaps not as significantly as

Table 1
Sensitivity of selected statistics to different initial random perturbations

Integration	$\mathscr{L}$	$\mathscr{D}$	Z_I	w_2	$\Delta\Theta_1$
CNTRL1	203.0	14.4	855	0.276	0.554
CNTRL2	221.6	12.2	864	0.307	0.395
CNTRL3	201.4	18.3	835	0.302	0.421
$\bar{x}$	208.7	15.0	851	0.295	0.455
σ_x	11.2	3.1	15	0.016	0.088

Table 2
Sensitivity of selected statistics to different initial random perturbations in non-drizzling ERM model without bin-microphysics and at two resolutions (regular and half resolution). Given are the mean values and standard deviations from three realizations of the flow

Integration	$\mathscr{L}$	Z_I	w_2	$\Delta\Theta_l$
CNTRL	213 ± 7	934 ± 4	0.391 ± 0.017	0.410 ± 0.065
HLFRS	216 ± 2	904 ± 4	0.337 ± 0.021	0.360 ± 0.038

might be expected. Variability among realizations is due to the fact that the integrations are not near a statistically steady state, so the ergodic hypothesis (which states that a time average can be substituted for an ensemble average) fails.

Owing to the significant variability among baseline simulations, subsequent comparisons are based on normalized statistics; namely, given some parameter x from an integration, we tabulate $x* = (x\text{-}\bar{x})/\sigma_x$ where $\bar{x}$ and σ_x are taken from Table 1. Consequently, $|x*|$ large implies that the difference between a simulation and the mean of the control runs is large relative to the standard deviation among the control runs. The results from all cases (including the control runs from which $\bar{x}$ and σ_x are derived) are given in Table 3. Because among the control experiments $|x*|_{\max} = 1.2$ we take values of $|x*| > 1.2$ to be a crude measure of significance.

3.3. Sensitivity to grid spacing

To explore the effect of resolution, we repeated a control simulation with double grid spacing in both spatial dimensions and time. This integration, designated as RES1 in Table 3, produced more drizzle, which significantly depletes the mean LWP, generates more stability across the cloud base, and, consequently, causes less entrainment. In an attempt to understand what produced this behavior, we returned the horizontal resolution to its original value. An examination of RES2 suggests that this mitigated the differences in $\mathscr{D}$ and Z_I but aggravated the departures in $\Delta\Theta_l$, w_2, and $\mathscr{L}$. Integration RES3, for which only the time step was doubled, produced results similar to RES2, although the increase in drizzle production was mitigated.

In retrospect it may not be surprising that the ERM integrations are sensitive to resolution. This is because the turbulent-closure scheme in the ERM is based on the properties of three-dimensional turbulence, for which there is an active cascade of energy to smaller scales. In dry ERM integrations, increasing the resolution tends to increase the strength of the turbulence as a result of a well known property of two-dimensional turbulence: the up-scale cascade of energy. The greater dynamic range in scales increases the separation between the energy-containing scales and the dissipation scales, and thus the energy dissipation balances its production at a higher level of turbulent kinetic energy.

For example, consider the dissipation term in the model. It comes from forming the inner product between the velocity and the divergence of the subgrid stress:

$$\overline{u}_i \frac{\partial \bar{\tau}_{ij}}{\partial x_j} = \frac{\partial\left(\overline{u}_i \bar{\tau}_{ij}\right)}{\partial x_j} - \epsilon, \text{ where } \epsilon = \bar{\tau}_{ij} \frac{\partial \overline{u}_i}{\partial x_j} = \frac{1}{2} \bar{\tau}_{ij} \overline{D}_{ij}, \tag{14}$$

Table 3
Normalized statistics from sensitivity runs

Integration	$\mathscr{L}$	$\mathscr{D}$	Z_{I}	w_2	$\Delta\Theta_1$	Comment
CNTRL1	−0.5	−0.2	0.2	−1.1	1.1	
CNTRL2	1.2	−0.9	0.9	0.6	−0.7	
CNTRL3	−0.7	1.1	−1.1	0.5	−0.4	
RES1	−2.5	4.2	−2.5	−4.6	2.7	$2(\Delta x, \Delta z, \delta t)_{\mathrm{cntrl}}$
RES2	−3.7	2.5	−1.1	−4.7	3.3	$2(\Delta z, \delta t)_{\mathrm{cntrl}}$
RES3	−3.8	1.4	−0.1	−4.3	2.8	$2(\delta t)_{\mathrm{cntrl}}$
TURB	−1.4	0.6	−0.4	−0.2	0.6	No diffusion for drops
SNDNG	−0.3	1.7	−0.3	−0.3	1.7	Moistened sounding

where $\overline{D}_{ij}$ is the deformation:

$$\overline{D}_{ij} = \left(\frac{\partial \overline{u_i}}{\partial x_j} + \frac{\partial \overline{u_j}}{\partial x_i} \right)$$

and $\bar{\tau}_{ij}$ is the subgrid stress (proportional to an eddy viscosity K_{m}):

$$\tau_{ij} = K_m \overline{D}_{ij} \text{ where } K_m \propto l_0^2 D \text{ and } D \equiv |\overline{D}_{ij}|,$$

and l_0 is a length scale proportional to the gridscale. These relationships imply that $\epsilon \propto l_0^2 D^3$, so that for a given flow, the dissipation is be constant as l_0 is changed (within an inertial range) so long as $D \propto l_0^{-2/3}$. In the inertial subrange of three-dimensional turbulence, this scaling relationship is satisfied [e.g., see (Lilly, 1967)]. For this reason resolved properties of three-dimensional flow should not change as resolution is increased so long as l_0 is held fixed —hence, l_0 is sometimes called the *filter scale*, which, with appropriate choice of constants, is usually made commensurate with the grid scale (Mason, 1994).

In two-dimensional turbulence, however, this is not the case. We have analyzed ERM simulations of radiatively-active smoke clouds (which mimic stratus clouds but eliminate the complications associated with phase changes) and found that D falls off much more rapidly with inverse length scale. For instance, if in analogy with three-dimensional turbulence we take $D^2 \propto k^3 E(k)$, where $E(k)$ is the energy density and k is an inverse length scale (i.e., $k \propto l_0^{-1}$), we get $D \propto k^{-1}$ (where for simplicity we have assumed [3] that the enstrophy range scaling for two-dimensional turbulence is $E(k) \propto k^{-4}$). This leads to less dissipation for a given flow as the resolution is increased. However, a steady-state can only be achieved if energy is dissipated at the same rate at

[3] Exactly how the energy spectrum scales is still controversial. On dimensional grounds one can argue that $E(k) \propto k^{-3}$, but this leads to a singularity which is removed through the corrected log scaling proposed by Kraichnan. Here $E(k) \propto k^{-3}[ln(k)]^{-1/3}$. Our choice of k^{-4} scaling is motivated by numerical experiments that tend to provide scaling laws with this exponent [see Bowman (1996) for a discussion of the current status of these issues]. Regardless, given energy-spectral-density scaling $E(k) = k^{-n}$, so long as $n > (5/3)$, the essence of our argument is unchanged.

which it is produced. Because in a *given* 2-D flow there is less dissipation for increased resolution, as resolution is increased, a quasi-steady balance between turbulent kinetic energy (TKE) production and dissipation can be formed only for larger values of TKE. However, given equal forcing, the above arguments indicate that simulations with finer resolution will have more TKE, because this is the only way for them to achieve the required rate of dissipation.

As a consequence of the above arguments, we expect w_2 to decrease as the grid mesh is made coarser [this is well illustrated by comparing the statistics of the ERM integrations (without drizzle) in Table 2], but, this leads to less entrainment. The fact that $\mathscr{L}$ is lower and $\mathscr{D}$ is larger (especially for experiment RES2) suggests that coarse vertical resolution leads to the more efficient production of precipitation. This is consistent with the fact that coarser vertical discretization in and of itself (as well as in combination with weaker turbulent circulations) results in smaller grid-averaged supersaturations (Clark, 1974; Stevens et al., 1996a). Consequently, fewer drops are activated, a broader spectrum is established, and the initial stages of drizzle production are enhanced. This effect appears to dominate the hypothesized decrease in drizzle production owing to the lack of turbulent support for drizzle-size drops in a weaker circulation (Feingold et al., 1996a). More drizzle, in turn, is a positive feedback, as it enhances the stabilization of the sub-cloud layer and suppresses w_2. These results suggest that ERM simulations are more sensitive to resolution changes when drizzle is active than otherwise suspected. Moreover, they highlight the direct and indirect effect that grid-mesh size has on microphysics. The direct effect is due to the increasing importance of subfilter terms as grid-size is increased; the indirect effect is due to the changing nature of the turbulent circulations themselves.

3.4. Sensitivity to turbulence-closure model

Above we discussed how the nature of the closure on the momentum-advection terms interacts with some fundamental properties of two-dimensional turbulence to impose a grid-mesh sensitivity on the flow. How does our choice of closure for the advective terms in the drop-distribution evolution equation impact the microphysical evolution of the cloud?

As discussed earlier, it may often be convenient to form a parameterization that models the collective effect of all subfilter processes (condensation as well as mixing). When this is done with a bulk-microphysical model, one can represent the subfilter processes [Denoted by $\mathscr{P}_{\psi}(\overline{\phi_i})$ in Eq. (7)] influencing the evolution of nonconserved quantities (such as q_{l} and θ) in terms of the turbulent flux of conserved quantities. Consequently, simple cloud models often partition the liquid-water into two bins: a cloud-water category q_{cw}, assumed to move with the turbulent motions and to adjust to the ambient saturation vapor pressure instantaneously, and a rain-water category q_{rw}, assumed to have sufficient inertia to be independent of small-scale motions. By so doing we can neglect the effect of subgrid motions on q_{rw} while representing the collective effects of subfilter processes on the cloud water field as $\mathscr{P}_{q_{\mathrm{l}}}(\overline{\phi_i}) = a(\overline{w'\theta_{\mathrm{l}}'}) + b(\overline{w'q_{\mathrm{t}}'})$, where a and b are thermodynamic constants depending on the mean state (Deardorff, 1976).

In bin-microphysical models such as ours, where the liquid water is partitioned into 25 size bins, no *qualitative* differentiation is made among bins. Liquid water in each bin is assigned a fall velocity based on the average size of the drops in that category, and every attempt is made to predict the finite phase-relaxation times of liquid water accurately. Although we can imagine developing a rather complex parameterization scheme that would account for phase-relaxation times and drop-inertial effects, it seems worthwhile to first ask how sensitive the results are to the details of the representation of the subgrid fluxes of liquid water. In sensitivity test TURB we set the subgrid fluxes of liquid water to zero for the duration of the integration; so doing indicates that the results are not obviously sensitive to the manner in which $\mathscr{P}_{q_l}(\overline{\phi_i})$ is specified. Nonetheless, caution should be used in interpreting this result: because energy has a hard time getting to the small scales in two-dimensional flows, the importance of the subgrid terms on the evolution of the microphysics may be underestimated.

4. Discussion

4.1. Comparing models to observations

In the introduction, we claimed that comparisons between models and observations can often be misleading. Here, we offer two arguments in support of this statement.

First, the amount of latitude allowed in comparisons between models and observations, at most, supports a conclusion that the model is not grossly inconsistent, but this conclusion is rarely drawn. The source of this latitude is multifaceted. On the one hand, scatter in the measurements is considerable. Many measurements are difficult to make, particularly in cloudy boundary layers. On the other hand, most observational data sets cannot specify the initial conditions with sufficient certainty to constrain the model. There is still no way to measure large-scale divergence (i.e., subsidence) with any confidence, nor can mesoscale effects be accounted for. Often cloud-base condensation nuclei are not measured simultaneously with cloud-drop measurements, and the radiative-flux profiles at cloud top are difficult to specify with any certainty. As a result, modelers have considerable latitude in choosing initial conditions (e.g., compare the CCN spectrum used by Bott et al., 1996 to the one used by Ackerman et al., 1994 to model the same case).

The effect of an uncertainty in initial conditions is accentuated in precipitating boundary layers. For illustration, we repeated the control integration of the previous section using a 1% moister sounding (i.e., the total water mixing ratio was uniformly increased by 0.1 g kg^{-1}). The results from the integration are tabulated under the SNDNG heading in Table 3. The increased water in the initialization significantly affects the amount of precipitation produced during the integration, which has a stabilizing effect on the subcloud layer. Given these systematic effects, the lack of significant change in w_2, Z_I and $\mathscr{L}$ is surprising. These changes are commensurate to those found when a slightly different kernel was used in the collection calculation. Admittedly this sensitivity signature may not exist in all situations, and it also may be biased by aspects specific to two-dimensional simulations. Nevertheless, our results suggest that matching simulated and observed surface-drizzle fluxes with any precision may be difficult.

Second, even if the model has significant inconsistencies in the microphysics, so long as the model is able to maintain approximately correct amounts of liquid water (a strongly constrained thermodynamic variable), reasonable dynamic responses will ensue for cases in which drizzle is not dynamically important. This fact has motivated the implicit treatment of the microphysics by many large-eddy simulations of stratocumulus. This implicit treatment of microphysics assumes that the cloud is at all times just saturated, thereby allowing one to express liquid water as a function of total water, pressure, and conserved moist thermodynamic variables (e.g., liquid-water potential temperature). Hence in nonprecipitating clouds, the detailed microphysical structure is relatively unimportant to the evolution of the layer so long as the LWP is reasonably well predicted. Consequently, matching mean thermodynamic profiles to observations does not provide a critical test of the microphysics.

What then is the best way to evaluate the suitability of the model to address various hypotheses? For reasons discussed above, to view the model as an initial value problem and to compare the model results at different times does not seem promising. If the view of the model as an initial value problem is not critical, how can we judge the merits of a particular model? A key advantage of a numerical model is that it can be controlled. Numerical experiments can be performed and hypotheses may be formed on the basis of these experiments. Cause and effect for a modeled process may be elucidated, and means of interrogating the physical system can be suggested. This advantage does not seem to be well appreciated. We believe that, apart from demonstrating that numerical models represent physical processes in qualitatively realistic ways, *less* attention should be devoted to addressing the fidelity of the model, partly because the model can never be a surrogate for reality (inevitably some of its predictions will be proved wrong) and partly because most tests of model fidelity are not sufficiently discriminating. Instead, physically consistent models should be used to suggest relationships which can then be looked for in the observations. In other words models are best used to state hypotheses, not answer them. To illustrate this point, we suggest three tests (below) for bin-microphysical models of the marine boundary layer, more as an outline for use than a means of 'verification'.

4.1.1. Drop concentrations and turbulent kinetic energy

A numerical model, such as the one discussed in the previous section, could be used to develop a prediction about the relationship among aerosol concentrations, turbulent kinetic energy, and drop concentrations. Stevens et al. (1996a) suggested a special case of this type of test wherein simultaneous measurement of CCN spectra, cloud-drop spectra, and vertical-velocity distribution functions are made and compared with results of simulations. Alternatively, one could use an ERM to predict an empirical relationship between scale-velocities (i.e., TKE or the buoyancy scale velocity w_*) and the velocity w_a, responsible for activating the average number of drops. This sort of prediction depends on the assumption that w_a is a unique function of TKE or w_*, but if such a relation exists in the model, it could then be compared with observations.

4.1.2. Drizzle rates as a function of large-scale parameters

By using an ERM one could make headway in mapping out a relationship between the drizzle flux at cloud base and parameters such as LWP, TKE, and cloud-drop concentrations. Feingold et al. (1996a,b) took the first step by illustrating that cloud-drop residence times are a critical parameter (in numerical models) for expressing the rate of precipitation formation; hence, both cloud depth and TKE (or some appropriately defined velocity scale) are important factors in controlling the rate of precipitation formation. These parameters could be used in a simple test to determine whether models with bin-microphysics are able to represent the conditions under which clouds form precipitation and the rate at which precipitation-size drops form.

4.1.3. Drizzle and boundary layer structure

By using an ERM we could hypothesize how precipitation affects the large-scale thermo-dynamic and turbulent structure of the stratocumulus-topped boundary layer. For instance, casual observations of the radar data from ASTEX suggest a relationship between cloud-top height and drizzle production. If the model can reproduce this relationship, it could predict circumstances favorable for such an effect (e.g., light drizzle and weak stratification, or strong drizzle and weak stratification) that could then be observationally tested. Models such as the ERM may also be used to develop relationships between drizzle rates and boundary-layer stratification, or drizzle rates and boundary-layer TKE which could, in principle, be tested by observations.

5. Prospects for one-dimensional models

Are we imposing a double standard when we question the use of one-dimensional models and endorse the use of two-dimensional models? First we must emphasize that our questions about the utility of one-dimensional models with binned microphysics are only relevant to current formulations of such models, and to processes, or hypotheses, *which depend strongly* on an accurate representation of cloud microphysical processes in a fully turbulent flow. We contend that results from the most sophisticated microphysical model will be of little use if the model is not properly coupled to the turbulence. Second, we must reiterate that the failure of one-dimensional models with binned microphysics to accurately represent the activation process at cloud base, leads to the production of an unrealistic droplet spectrum. This is a qualitative and quantitative failing of such models. In contrast, we argue that inconsistencies in the closure assumptions used in two-dimensional models leads to results which can be quantitatively unreliable. However, a number of papers have shown that two-dimensional models can at least qualitatively represent the rudimentary aspects of the cloud microphysical structure. As a result we believe that we are being fair when we say that in contrast to current formulations of their one-dimensional counterparts, two-dimensional models can be useful, if only in a qualitative sense. Note that this outlook should not be used to support the conclusion that two-dimensional models are *necessarily* better than their one-dimensional counterparts. Indeed, recent intercomparisons of simulations of a radiatively driven smoke cloud (Bretherton et al., 1997; submitted to the Quarterly

Journal of the Royal Meteorology Society) suggest that the relationships between velocity variances and entrainment in one-dimensional models more closely resembles the three dimensional result than what is predicted by two dimensional models.

What then are the prospects for one-dimensional models? For processes that require detailed representations of drop and aerosol interactions and that operate on the timescales of days, the use of a multi-dimensional model seems out of the question. Because such questions seem so well suited to one-dimensional models, [4] we argue that adequate parameterizations need to be developed for the one-dimensional models. The use of detailed three dimensional models would seem well suited to this task. If adequate parameterizations can be developed, then the prospects of using one-dimensional models (to study the interactions between drops and aerosol in turbulent boundary layers over long timescales) seems bright.

Although we have focused our discussion on a rather narrow class of models it is worth pointing out that our criticisms of one-dimensional models are equally appropriate to any model that couples a detailed representation of cloud microphysical processes to a dynamical framework which under-resolves the cloud-eddy-scale circulations. In recent years, microphysical models with increasing levels of sophistication have been incorporated into mesoscale and general circulation models. Our analyses suggest that if these models are to be physically based, then they need to explicitly account for sub-grid scale processes such as turbulence structure and inhomogeneity and its interaction with microphysical processes such as activation of CCN, sedimentation of hydrometeors and collection. Thus further development of the one-dimensional models would be instrumental in paving the way for a better representation of microphysics in larger scale models.

6. Summary

A number of questions regarding the effect of aerosols on boundary layer clouds remain unresolved. To begin addressing these questions it is tempting to use bin-microphysical models coupled to models of turbulence. In this paper we critically evaluate the merits of two outstanding candidates for such an approach: Bin-microphysical models coupled to either one- or two-dimensional dynamic models.

Two groups (e.g., Ackerman et al., 1993, 1994, 1995; Bott et al., 1996) have already coupled binned representations of aerosol and cloud-drop microphysics to one-dimensional turbulence-closure models. Using such a model, Ackerman et al. (1993, 1994) began addressing a number of hypotheses which relate to how aerosol cloud-drop interactions may effect the radiative properties of clouds, or the steady state distribution of aerosols. Their approach is questionable because it ignores important forcing terms,

[4] Although one wonders whether a crude parameterization of microphysics in a sophisticated turbulence model is more (or less) realistic than a sophisticated microphysical model coupled to a crude parameterization of turbulence.

specifically, the subfilter-scale correlations in the aerosol-mass spectrum and the supersaturation spectrum, which are responsible for activating drops at the cloud base. Their neglect of these terms leads to a curious representation of the cloud, one in which most drops are activated at cloud top and then are diffused toward cloud base. Although this problem is probably the dominant error we also point out that the neglect of sub-filter correlations in the collection equation may significantly retard the initial growth of a drizzle mode in the layer-averaged drop concentrations–although once a drizzle mode is established, further growth of this mode is not significantly retarded by the neglect of inhomogeneities on the large-eddy scale.

The use of a two-dimensional framework as a host for the microphysical model is appealing because it allows a rough representation of the energy-containing eddies in buoyancy-driven flows. This approach has been used to study the microphysics of stratocumulus clouds, and it often produces realistic-looking cloud fields. Stevens et al. (1996a) show that the distribution of in-cloud trajectory timescales is predicted to be narrower in a two-dimensional model than in its three-dimensional counterpart. Here we show that the closure assumptions typically used in two-dimensional models do not promote convergence as the grid spacing goes to zero. Rather, the energy-containing scales become more and more energetic. The inviscid system may well be the limit of two-dimensional flows, although further work is needed to establish this. Both the problem of closure in two dimensions and the problem of trajectory timescales relate to the lack of an energy cascade in two dimensions, which implies an accumulation of energy on the larger scales. The fact that the model is quite sensitive to resolution (in space and time), but not sensitive to the closure assumption for the condensate may also be an artifact of two dimensions. Nonetheless, in some important respects results from two-dimensional models resemble both reality and three-dimensional simulations (at least for the case of buoyancy-driven flows). For this reason they may be a good choice for assessing qualitative relationships in a narrow range of parameter space, but they have limited value when more quantitative understanding is desirable.

In addition to the sensitivities inherent in its formulation the ERM integrations indicate that results are less robust in the presence of drizzle than they are in the absence of drizzle. Moreover, small changes in the representation of the collection probabilities or total water content may lead to significant changes in drizzle production, which can be reflected in a number of other statistics. These sensitivities of the model, in combination with its poorly formulated closure and the difficulties in making sufficiently precise atmospheric measurements, make models, such as the ERM, difficult to constrain observationally (i.e., the precision which with we would like to be able to say they work well is not at the moment attainable). For this reason we suggest a way of using the model that relies less on it being true (clearly, it is not), and more on it as a useful tool for suggesting new ways of interrogating the observations.

Acknowledgements

The simulations reported on in this paper were conducted while BS was a student at Colorado State University. His participation in this work was make possible by NASA

Graduate Fellowships on Global Change (grants NGT-30231). GF and WRC are thankful for the support of the National Science Foundation (USA) grant ATM-9529321. The boundary layer working group of GCSS is also thanked for their collective insights and cooperative spirit, all of which has influenced the authors (and their ideas) in the most positive of ways.

References

Ackerman, A.S., Toon, O.B., 1996. Unrealistic desiccation of marine stratocumulus clouds by enhanced solar absorption. Nature 380, 512–515.

Ackerman, A.S., Toon, O.B., Hobbs, P.V., 1993. Dissipation of marine stratiform clouds and collapse of the marine boundary layer due to the depletion of cloud condensation nuclei by clouds. Science 262, 226–229.

Ackerman, A.S., Toon, O.B., Hobbs, P.V., 1994. Reassessing the dependence of cloud condensation nucleus concentration on formation rate. Nature 367, 445–447.

Ackerman, A.S., Toon, O.B., Hobbs, P.V., 1995. A model for particle microphysics, turbulent mixing, and radiative transfer in the stratocumulus-topped marine boundary layer and comparisons with measurements. J. Atmos. Sci. 52, 1204–1236.

Belyaev, V.I., 1967. The evolution of the droplet condensation spectrum in clouds. Izv. Acad. Sci. USSR, Atmos. Oceanic Phys. 3, 341–343.

Bott, A., Trautmann, T., Zdunkowski, W., 1996. A numerical model of the cloud-topped planetary boundary layer: radiation, turbulence and spectral microphysics in marine stratus. Q. J. R. Meteor. Soc. 122, 635–667.

Bowman, J.C., 1996. On intertial-range scaling laws. J. Fluid Mech. 306, 167–181.

Chen, C., Cotton, W.R., 1987. The physics of the marine stratocumulus-capped layer. J. Atmos. Sci. 44, 2951–2977.

Clark, T.L., 1974. On modelling nucleation and condensation theory in the Eulerian spatial domain. J. Atmos. Sci. 31, 2099–2117.

Clark, T.L., Hall, W.D., 1979. A numerical experiment on stochastic condensation theory. J. Atmos. Sci. 36, 470–483.

Cooper, W.A., 1989. Effects of variable droplet growth histories on droplet size distributions: Part I. Theory. J. Atmos. Sci. 49, 1301–1311.

Deardorff, J.W., 1976. Usefulness of liquid-water-potential temperature in a shallow-cloud model. J. Appl. Meteor. 15, 98–102.

Feingold, G., Stevens, B., Cotton, W.R., Walko, R.L., 1994. An explicit cloud microphysics/LES model designed to simulate the Twomey effect. Atmos. Res. 33, 207–234.

Feingold, G., Stevens, B., Cotton, W.R., Frisch, A.S., 1996a. The relationship between drop in-cloud residence time and drizzle production in numerically simulated stratocumulus clouds. J. Atmos. Sci. 53, 1108–1122.

Feingold, G., Kreidenweis, S.M., Stevens, B., Cotton, W.R., 1996b. Numerical simulations of stratocumulus processing of cloud condensation nuclei through collision-coalescence. J. Geophys. Res. 101, 21391–21402.

Feingold, G., Walko, R.L., Stevens, B., Cotton, W.R., 1997. Simulations of marine stratocumulus using a new microphysical parameterization scheme. Atmos. Res., this issue.

Kogan, Y.L., Lilly, D.K., Kogan, Z.N., Filyushkin, V.V., 1994. The effect of CCN regeneration on the evolution of stratocumulus cloud layers. Atmos. Res. 33, 137–150.

Kogan, Y.L., Khairoutdinov, M.P., Lilly, D.K., Kogan, Z.N., Liu, Q., 1995. Modeling of stratocumulus cloud layers in a large eddy simulation model with explicit microphysics. J. Atmos. Sci. 52, 2293–2940.

Lilly, D., 1967. The representation of small-scale turbulence in numerical simulation experiments. In: Goldstine, H.H. (Ed.), Proceedings IBM Scientific Computing Symposium on Environmental Sciences. IBM Form No. 320-1951., pp. 195–210.

Martin, G.M., Johnson, D.W., Rogers, D.P., Jonas, P.R., Minnis, P., Hegg, D.A., 1995. Observations of the interaction between cumulus clouds and warm stratocumulus clouds in the marine boundary layer during ASTEX. J. Atmos. Sci. 52, 2902–2922.

Mason, P., 1994. Large eddy simulations: A critical review of the technique. Q. J. R. Meteor. Soc. 120, 1–26.
Moeng, C.-H. et al., 1996. Simulation of a stratocumulus-topped PLB: Intercomparison among different numerical codes. Bull. Am. Meteor. Soc. 77, 261–278.
Stepanov, A.S., 1976. Influence of turbulence on the size spectrum of cloud drops during condensation. Izv. Acad. Sci. USSR, Atmos. Oceanic Phys. 12, 281–292.
Stevens, B., 1996. On the dynamics of precipitating stratocumulus. Ph.D. dissertation, Colorado State University, Ft. Collins, CO 80523, USA, p. 140.
Stevens, B., Feingold, G., Cotton, W.R., Walko, R.L., 1996a. Elements of the microphysical structure of numerically simulated nonprecipitating stratocumulus. J. Atmos. Sci. 53, 980–1007.
Stevens, B., Walko, R.L., Cotton, W.R., Feingold, G., 1996b. The spurious production of cloud-edge supersaturation by Eulerian models. Mon. Wea. Rev. 124, 1034–1041.
Telford, J.W., 1996. Clouds with turbulence; the role of entrainment. Atmos. Res. 40, 261–282.
Twomey, S.A., 1959. The nuclei of natural cloud formation: Part II. The supersaturation in natural clouds and the variation of cloud droplet concentration. Geofisica Pura e Applicata 43, 243–249.